The Arithmetic Teacher

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Arithmetic—Friend or Foe?
FRANCIS R. BROWN

Developing Creative Thinking in Arithmetic
CALVIN H. REED

Bibliography of Mathematics Books for Elementary School Libraries
ADRIEN L. HESS

Meanings in Division
H. C. CHRISTOFFERSON

Group Methods in Primary Grades
ANTHONY J. GRECO

THE ARITHMETIC TEACHER

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THE ARITHMETIC TEACHER

Volume IV Number 1

February 1957

Arithmetic—Friend or Foe?*

FRANCIS R. BROWN
Illinois State Normal University, Normal, Ill.

WIDESPREAD CONCERN over the shortage of engineers and technicians has now focused attention on what the Carnegie Corporation of New York calls a national weakness: namely, the mathematical incompetence of the average—and even the above-average—American.

A study financed by the corporation and carried out by the Educational Testing Bureau of Princeton, New Jersey, has analyzed some of the reasons why so many students at all levels who do not avoid mathematics like the plague, either fail or just manage to scrape through. The answer they give, in just two words, is POOR TEACHING. An assumption is then made that this poor teaching is due, in large measure, to the lack of an adequate background by the teacher. One of the findings, for instance, was that although all states require courses in *education* for teachers of mathematics in secondary schools, a third of the states require no *mathematics* for certification of secondary school mathematics teachers. In the elementary schools, the picture is even worse. Teachers are certified as mathematics teachers who never had any college work at all in mathematics—or for that matter, some have had no work in mathematics beyond the elementary school.

In reviewing these findings, the Carnegie Corporation says: "Under such circumstances it is no surprise that one professor states: 'Elementary teachers, for the most part, are ignorant of the mathematical basis of arithmetic'; and another professor adds about secondary-school mathematics teachers: 'They are not as good as our run-of-mine juniors'" (*Time* 2-27-56). Are these justifiable conclusions? What per cent of the teachers just meet certification requirements? Does this imply that a study of subject matter is all that is needed to produce a teacher? Regardless of the implications—fair or otherwise—this statement has been read on a national basis.

The survey *does* indicate that teachers have had no real understanding of the subject, and no zeal for teaching it—present company of course excepted!

It is becoming widely recognized that the making of a future scientist begins in grade school when the youngster first learns about numbers. Mathematics is the backbone of the sciences. Without a firm grounding in the third of the 3 R's, a pupil will never make the grade as a scientist or engineer.

In a recent address at the Association of Higher Education, the former Senator Dr. William Benton, now editor of the Encyclopedia Britannica, pointed up vividly one aspect of the world situation in the

* From an address given at the Western conference of the Illinois Council of Teachers of Mathematics at Canton, Ill., March 24, 1956.

training of engineers. The Communists have already passed the Americans in the yearly production of engineers and mathematics and science teachers. As a basis for this, they have created a ten-year school (six days a week, ten months a year), that will be compulsory for all by 1960. The first four years concentrate on reading, writing, arithmetic, and Russian—perhaps this could be called the 4 R's of Russia. Our own 4 R's now, according to Dr. Conant, of Harvard, are Reason, Resourcefulness, Responsibility, and Relations (Human). In the last six years of the Russian compulsory system, more than 40 per cent of the students' time is given to science and mathematics, including algebra, geometry, and trigonometry, plus four or five years of physics, four years of chemistry, two years of biology, a year of astronomy, and a year of psychology. They also take six years of a foreign language—English is the favorite, because it is now recognized as the language needed to study science.

Last year, for all the high schools of the United States, we produced only 125 new physics teachers! Most of the pressure to do something about this failure—that is, of attracting and educating able pupils—is directed at the high schools; but I agree with those who think that the problem *must* be tackled sooner. It is in the elementary school that the basic understanding of numbers is formed. And it is here that the pupil learns either to love or to hate mathematics. Few children remain neutral.

The remainder of this talk will be concerned with two aspects of attacking this problem of developing pupils who have confidence in and understanding of basic mathematical concepts and skills.

Able and Confident Teachers

The first aspect—and, to me, the most important—is to develop able and confident teachers. At Illinois State Normal University, the prospective elementary school teachers study a course in the freshman year entitled "Basic Concepts of Arithmetic." I have two basic objectives for this course: the first, that the students leaving the course understand the meaning and use of the fundamentals of elementary mathematics; and second, that they like mathematics. To help meet the individual differences of the students, we offer two levels of this course: one for those who are advanced in ability and/or background, and one for those with a less mature background. The final sectioning depends largely upon scores made on a test of arithmetic computation and reasoning given the first day of class; but the initial sectioning is based on oral questioning. One of the questions asked is, how much mathematics have you had; and a second question is, how do you like mathematics? The reaction to this second question has been a good indicator in placement, as the error of original placements for twelve sections has been less than 10 per cent in spite of the fact that the majority of the students have spent the same number of years studying mathematics.

Does it bother you to know that young minds will be exposed to the study of mathematics by teachers who openly dislike the subject? A question I ask the first day of these prospective teachers is: "How would you like for your son or daughter to have a mathematics teacher who has the attitude and background that you have?"

In a survey reported recently by W. H. Dutton, it was found that among a group of prospective teachers of arithmetic, 74 per cent responded with an unfavorable attitude toward arithmetic and a large number of these responses used language that was excessive and emotional. Think of it: the ratio of unfavorable response to favorable response was approximately 3 to 1. These people were formerly pupils in our public schools, acquiring the same background as many, many others. Are we going to recognize this problem of dislike toward mathematics—with the usual consequences of reduced efficiency in its use—without doing something about it?

Are we to send out teachers with this feeling of fear and hate?

There seems to be little question that the teacher plays a prominent part in the learning experience of the pupils. A study was made of motivating devices, from the standpoint of the student. Fifty-four high school and 93 college students were asked, after various lessons, how they were motivated. The largest group (32 per cent) indicated that the students had reacted to "competition, rewards, and prizes." The second largest specific category (19 per cent) showed that students reacted to the teacher's personality and knowledge of subject matter, rather than the motivation which the teacher thought he used. The third category was visual aids (18 per cent). A closer examination of the students' replies showed that—no matter what they said—the teacher's personality was an important factor in the motivation of the lesson. As we work with others, we must recognize that we teach by what we say, what we do, what we are. A teacher never knows what he has taught until he tries to "unteach" it.

What does this lead up to? That our first attack on the problem is to strive toward a teaching corps that understands arithmetic and likes it. All of the innovations in methods and teaching aids will be of little avail if the teachers aren't sold on their work; if they aren't striving to do a good job.

An editorial in the March 1, 1956 Chicago *Daily News* says: "Now that this weakness (poor teaching) has been exposed and the causes (teachers with no real understanding of arithmetic and no zeal for teaching it) at least partially understood, perhaps we can set about creating more mathematics enthusiasts on our teaching staff in the sure knowledge that their enthusiasm will be catching."

We must reach the teachers through meetings such as this. The ICTM is attempting to reach more and more people. The membership is reaching upwards to 800, but it should be many more. With strength of unity of purpose, we can devise means of helping teachers overcome fears and fill in the gaps in their arithmetic background.

Every teacher of arithmetic should read THE ARITHMETIC TEACHER, a new journal especially created to bring to the classroom teacher (among others) the changes in the teaching of arithmetic that have taken place—and to report significant practices and problems of the classroom teacher. The reception of this journal has been such that within two years after the first issue (1954), it was announced that there would be an increase from four issues a year to six issues. Let me add that secondary school teachers are also finding this journal of significant help.

By attending conferences such as this and reading journals such as THE ARITHMETIC TEACHER, it is possible to evaluate better the work you and your schools are doing. It is safe to say that at present 90 per cent of the school program is a result of individual philosophy, practical needs, cultural demands, and trial and error teaching. There is a need for an understanding of research that has been reported. There is need for "action-research" to be carried on in your classrooms.

Research Sheds Light on Teaching

Action-research is planned to be carried out in local schools during the regular year through the cooperation of the local staff and other leaders who can help plan and carry out the research. The need for this is evident from replies of administrators to a questionnaire concerning their opinion and use of research. Sixty-five per cent of the administrators said that the present research was unsatisfactory, and 79 per cent claimed that it was foreign to school situations.

But research has had a very noticeable effect on the teaching of arithmetic. Many of the changes in teaching methods evidenced in new material coming to the classroom teacher is based on research. The persons interested in learning more

about these trends might secure for 50 cents a bulletin entitled: "What Does Research Say About Arithmetic?" published by the Association for Supervision and Curriculum Development; also a bulletin, "What Research Says to the Teacher Teaching Arithmetic" published by the Department of Classroom Teachers, 25 cents.

At this time I wish to mention just two of the studies that should influence the teaching of mathematics. The first of these was reported by Luchins in 1942. In a carefully planned study, Luchins proved that mechanized procedures for solving problems aided *only* in the solution of a particular type of problem, and that these same mechanized procedures became an impediment in problems that deviated from the type. In other words, *we do not learn to solve problems by rules.* How do *you* teach problem-solving? Are you successful? Do you have pupils who say: tell me what to put down and I'll solve the problem? Do you emphasize that the purpose of problem-solving is to learn to analyze problem situations, not just to secure an answer? With this purpose in mind, one assigns one problem to find five different solutions instead of five problems with one solution repeated. By this method both the teacher and the pupil are challenged.

The second study was reported in 1953 by Brownell, probably the outstanding research man in the field of arithmetic. He reported on the effects of practicing a complex arithmetic skill upon proficiency in its constituent skills. The experiment involved the study of division since this requires such prerequisite skills as knowledge of the basic facts in the four fundamental operations, proficiency in simple multiplication including carrying, proficiency in subtraction, without and with borrowing, and proficiency in simple division to the extent of knowing the process. His findings included:

1) Practice in division by 2-place numbers had no single, uniform, and predictable result so far as proficiency of sub-skills is concerned. In a given class there were effects both helpful and harmful and sometimes in the same child the effects were helpful in some sub-skills and harmful in others.

2) Children with the lowest degree of proficiency in the sub-skills made relatively little improvement therein while working on the new complex skill. For such children, planned remedial teaching is to be recommended. Perhaps it has been comforting to say to yourself, "Well, Johnny doesn't understand this very well, but he will soon get more practice in the advanced work. He will know it then." Will he? Brownell says the chances are that he won't. You might better find out his difficulty and give the necessary help so that he *can* complete the new work.

Research does help answer questions. Let me encourage you to do some experimenting in your work. Plan carefully and then carry through. This doesn't have to be a long experiment—may be a new technique to present a troublesome topic, a new type of assignment as a new approach to problem solving.

Once teachers begin to understand their subject and have a source of information and an incentive—often inspired by others in the school system—strides have been taken in solving the problem of poor teaching. Don't be a pessimist about this. You know what a pessimist is?—A pessimist is one who is always turning off the light to see how dark it is.

Teachers must not be like the young chap who went looking for something to do one afternoon. As he strolled through the park he noticed some rowboats for rent, so he decided that this was for him. As he drifted out into the stream he noticed that others were having a good time, but they were working rather diligently with the oars. This work didn't appeal to him, especially not at this time, as it was so pleasant just to put the oars in the boat and drift along. The day-dreaming youth enjoyed himself until he suddenly became aware of the noise that

came from the cataracts. But by this time he realized that he had drifted too long when he should have applied the oars.

Teachers must accept the responsibility of leadership and they must realize that they can no more give what they don't have than they can return from where they have not been. Review your reasons for teaching mathematics. Those who consider it merely a tool probably feel satisfied with teaching by drill. They start teaching addition facts as *facts* early, use flash cards and other drill techniques early, and seem to move quickly into the teaching program of basic facts and skills. This system has been found to provide inadequate results and in later years one becomes quite concerned with the lack of self-direction, lack of confidence, and incompetence on the part of the pupils. There is a heavy, and usually unsatisfactory reteaching program in the middle and upper grades. The initial gains in time are lost—and often so is the pupil. Pupils who have successfully but without meaning manipulated symbols, have records of good grades but in future years they fail to retain the skills and have no desire to use them.

Could you in the near future help organize a group of local teachers to discuss arithmetic and the problems of teaching it? Small discussion groups can be very beneficial. Try one if you care to face the challenge! Ask your superintendent or county superintendent for clinics or workshops. Organize pertinent questions for your work and secure help if needed from the Illinois Council of Teachers of Mathematics or from the teachers' colleges. *Think—read—discuss—experiment—evaluate.* You are responsible for producing pupils competent in mathematics, who do not hate it. To carry this out, be sure that you are a member of the teaching corps that understands arithmetic and likes it.

As you are striving to improve your methods of teaching arithmetic it is wise to keep the parents informed of the procedures you are using. Many of these parents are critical of new methods unless they understand something about their use. Preparing an Arithmetic Bulletin either as one small bulletin or as a series of short articles has been found to be interesting and effective for the parents and the teacher. This also stimulates the staff to clarify its own philosophy of teaching arithmetic. For other suggestions concerning contacts with the parents, see Esther Swenson's helpful article, "An Ounce of Prevention," in the February '56 issue of THE ARITHMETIC TEACHER.

Why Dislike for Arithmetic?

We will now move to a second aspect of the problem of producing pupils who have confidence in using mathematics. Dutton in his survey found the reasons that students' dislike of mathematics clustered around four points, namely:

a) lack of understanding;
b) dissociation of mathematics from life situations;
c) pages of word problems; and
d) fear of making mistakes.

Certainly the teacher is challenged to overcome these complaints. Suppose we examine them in more detail now.

These complaints come from all grade levels, but especially at those points when pressure is applied for immediate recall of facts. As a group, the primary youngsters love to do arithmetic activities, but the number that like it decreases in the second or third grade when basic facts are to be memorized. There is another significant loss with the work with fractions and another one with percentage.

This build-up of uncertainties and fears due to lack of understanding of quantitative situations is damaging to the personalities of the pupils. Growth in desirable personality traits is brought about by *successfully* overcoming frustrations. One of the major weaknesses in the teaching of arithmetic is the failure to develop pupils who know *when* as well as *how* to use the arithmetical facts and skills they study. These pupils, when faced with a frustrat-

ing situation, too often withdraw from the challenge.

We have all heard these complaints in one form or another. Does this generality of complaint suggest that at least part of the solution is concerned with the method of teaching? I think that it does. I propose that the use of a discovery or developmental approach will tend to alleviate each of the four complaints mentioned: namely, a) lack of understanding; b) dissociation of mathematics from life situations; c) dislike of pages of word problems; and d) fear of making mistakes.

I am not saying that method alone is the solution. Teachers must know and understand the subject materials. As yet there is no body of research to back up the use of the discovery method as a total teaching program, but isolated bits have been affirmative—and from teachers and pupils have come subjective approval in terms of confidence in the understanding, enjoyment, and use of arithmetic in solving problems.

Try the Discovery Method

For purposes of discussing the developmental or discovery method as I advocate it, I will use three classifications of procedures.

1) *Discovery by the pupil of the solution of a problem situation containing a new skill or concept is based upon previous experiences and knowledge.*

This emphasizes the sequential nature of arithmetic. Readiness is conceived as the prerequisite background of the individual so that he can solve a new type of problem with previously learned methods. The attainment of readiness is a continuing process of becoming *more ready* than one was previously. Readiness is specific; that is, readiness for a particular area. Thus, a multiplication situation is presented. The pupil can solve the problem by such procedures as adding, counting, making diagrams, or using various teaching aids. The concept of multiplication can then be developed as an economical method of solving the problem. But even the slower learner will have confidence of knowing some way of getting the solution from the beginning of the study. The thrill of discovery and the sense of satisfaction of getting the correct solution are needed by everyone. To aid teachers to sense this need for *some* method of approach to a new problem and then the thrill of success, I have them do some thinking with numbers expressed to bases other than 10. Thus, 4 plus 3 doesn't always equal 7; it may equal 12 if the base is 5. Then what does three 4's equal using base 5? To answer this one, the students use methods other than multiplication at first. If base 5 were to be used for all computations, it would be most economical to discover the basic facts and *then* memorize them—not vice versa.

It is worthwhile to note here that all the abstractions in arithmetic reside in the fact that our number relationship springs from a number system. In order really to understand these relationships one must understand thoroughly how the number system operates. One of the most significant modern emphases in the teaching of arithmetic is that all four fundamental processes are basically processes of re-grouping. Thus, adding 7 and 8 doesn't give more than was present at first, but there is a re-grouping into 10 and 5. This grouping is universally used, and economical for thinking, so the pupil is advised (and forced!) to learn it. Understanding this re-grouping relationship, a pupil presented with a division situation can find the solution by continued subtraction. Thus, he can work a new type of problem by using techniques and skills already known. Later, the meaning and use of division as a process will be clarified.

Arithmetical concepts are abstractions. These abstractions are based on sense experiences—sight, touch, sound. Thus, visual aids play a major role in discoveries by the pupil: not only during the primary years, but throughout the development of

new concepts. A pupil learning about area might study the problem of finding the number of tile blocks to cover a floor. He might proceed by direct measurement, especially if the floor is small and the blocks rectangular. He might discover the value of using an indirect method of measuring the length and width if the room is large. Area, then, is the amount of surface enclosed. To measure it, it is necessary to define a unit of measure. In general, a square unit is easier to use than some other unit. The area isn't feet times feet: it is the equivalent number of square units that can be placed on the surface. Furthermore, one square foot is not the same as a one-foot square. The relationship between these is by definition. A foot-square is a geometrical figure. A square foot is an abstract concept of a unit of surface.

2) *A second procedure in the discovery approach is group discussion of the different solutions to a given problem.*

In my experience, the majority of the students in the beginning mathematics course for elementary teachers are surprised and amazed that there are more than *the* solution for a given problem. The purpose of problem solving is to learn to analyze the situation—not just find an answer. The challenge is to recognize in the given problem situation a similarity with previously understood situations.

The discussion of various approaches in solving a problem such as how to add fractions with unequal denominators increases the pupil's understanding of the process. The language used by each pupil as he explains a solution is a good indication of his grasp of the topic.

Teachers cannot carry on this procedure if they do not understand arithmetic, but if they are willing to study—with their experience and background—they can evaluate the variety of pupil responses. There will be a greater emphasis on oral arithmetic. This will help improve understanding and will also help dispel the fear of making errors. This becomes real learning instead of merely memorizing, and the pupils work *with* the teacher instead of *for* him.

You will have gathered that the purpose of the textbook in this approach is altered somewhat. It still is a principal teaching aid. It can be used to check for most efficient procedures, after the pupils have had time to experiment. It is used for problem material and it furnishes carefully organized drill material. Drill is used—after the meaning has been established.

3) *A third procedure concerning the discovery approach is the organization of the work period so that each pupil can work at his own maturity level.* At the same time he is encouraged to move into more mature methods as soon as possible.

This problem of individual differences brings to mind the story of the shy young salesman who timidly approached the desk of the go-getter sales manager and in a scared little voice muttered, "You don't want to buy any insurance, do you?" "Certainly not," was the brusque rejoinder. "I was afraid you wouldn't," murmured the embarrassed young man, as he edged toward the door. "Just a minute!" exclaimed the exasperated sales manager. "You are positively the worst salesman I have ever seen in all my years of dealing with them. You must have confidence—confidence! Just to show you what I mean, and to give you confidence that you *can* make a sale, I'll buy a $10,000 policy from you." "Oh, thank you very much." "And now," went on the sales manager, signing the application for the policy, "you must learn some good techniques and *use* them." "Oh, I have," returned the salesman. "I have an approach for almost every type of businessman. The one I just used was my standard approach for sales managers."

With this principle in mind—each pupil working at his own level of maturity—the teacher sees that teaching aids are available—perhaps kept at the pupils' desks as with fraction materials, or in an arithmetic corner as with the bead counter,

abacus, pocket chart, flannel board, measuring equipment, etc. The teacher can then move among the pupils, encouraging, making necessary suggestions, and working with small groups. Those pupils who have need to use crutches such as the carry number in addition are allowed to use them. They will be encouraged by various means to drop them as they become capable of working meaningfully without them. They will be encouraged to use concrete materials at first and then to move on to the use of abstract symbols—but too often teachers stop at this stage. If pupils are to be able to use these arithmetic abstractions, they should be guided in the uses of them in concrete situations. In other words, the work proceeds from concrete to abstract to concrete. The work starts with problems, moves to the study of the mathematical skills, and then back to problems.

Throughout the study the pupils are encouraged and aided in making generalizations and recognizing the structure of number relationships. Without structure and organization, the limitless facts overwhelm even a superior mind.

The discovery approach does aid in developing confident and competent pupils who enjoy working with quantitative situations. Accuracy and understanding are stressed. But there are some problems, also. In the first place, the teacher must be alert and competent as she directs the learning without dictating the procedures. Allowing pupils to come up with unique solutions certainly challenges the teacher to evaluate the results. Progress in subject matter will seem comparatively slow at first. Teachers will need to evaluate carefully the many teaching aids. Don't use an aid just because it is labelled an aid. Check to see if the aid can do anything that cannot be done better by some other means.

One of the major problems is meeting individual differences. This discovery method will aid with the problem as the pupils work at their own maturity level on a particular topic. New topics are introduced to the entire group at the same time. The differentiation thus comes within the topic instead of acceleration. This is a recommendation for the lower and intermediate grades, but for outstanding pupils in the junior high school it may be better to accelerate into advanced work instead of spreading within the social topics. Social utility is a desirable aspect of mathematics education, but it is not mathematics. Those pupils who are capable of doing more abstract work should be allowed to make early progress toward the background leading to work in science and mathematics.

However, individual differences will remain a definite problem. One of the difficulties is illustrated by a little story. A parent visited an arithmetic class one day. In the back seat sat a small boy who had considerable trouble following the classwork. The teacher continued to call on him, attempting to guide his thinking. The visitor was surprised, during the discussion which followed, that the other pupils continually referred to the boy as "Sanka." At the end of the period, as the students left the room, the visitor's curiosity got the best of him, and he said to the teacher: "This was a very interesting period. But one thing does have me confused: that boy in the back seat—that's Frankie Allen, but the other students kept calling him 'Sanka.' Would you mind telling me why?" "Well," the teacher replied, "surely you know what Sanka is? It has had 85 per cent of the stimulant removed from the bean!"

Evidence is continuing to accumulate that teachers and pupils working together by this method are developing confidence as well as ability. You can't make the change overnight, nor will the first results be miraculous—but no matter what mathematics you teach, you will find it stimulating, challenging—and fun.

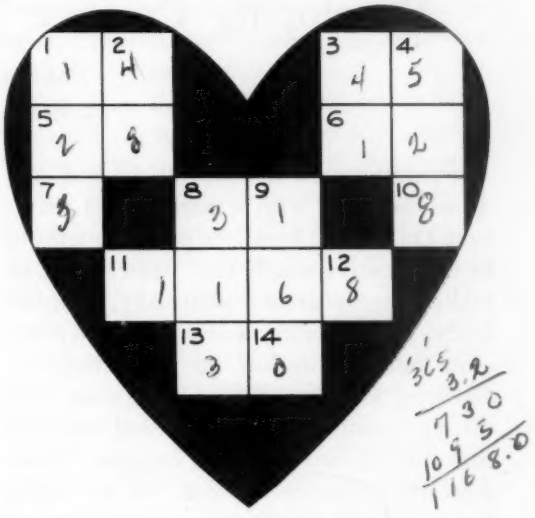
In conclusion, I wish to emphasize that we are in a period of revolutionary

changes in the teaching of arithmetic. With the change in method we are seeing changes in content. Formerly the arithmetic was presented in one book. Now we have eight or more. The skills have been analyzed to sub-skills and these carefully presented in sequential order. The child often gets lost or loses interest before he even acquires the basic skill itself. I predict that we will see a new organization with the basic skill presented earlier followed by practice on those sub-skills by the individuals who need them. There will be more emphasis throughout the grades on the understanding of basic ideas, such as numbers, symbols, form, and problem-solving. The content of the junior high grades needs careful re-evaluation.

The job is challenging. We are all involved. As we now proceed to our group meetings and then to our own jobs, may I remind you of what Emerson once said: "Nothing great was ever achieved without enthusiasm."

EDITOR'S NOTE. Professor Brown is worried about placing our pupils in the hands of teachers who know little about the mathematical background of arithmetic and who also have a dislike for the subject. But, he has some worthwhile positive suggestions for remedying the situation. To many other teachers it seems a bit strange that anyone should be fearful of arithmetic when it is an area of an infinite variety of interest and challenge. It is sad but true that many adults condition youngsters indirectly through their own fears of anything mathematical. It is also true that many adults, who memorized well as children and received good marks in school, are, as adults, very inept in a mathematical situation. Clearly, something is wrong with the subject, its organization, or its teaching, or a combination of these. Professor Brown suggests the method of *discovery* as a new view and approach to learning. It is also a good method for teachers to use in their thinking about their job of teaching. Why not have more good in-service meetings trying to *discover* how children best learn the kind of arithmetic that is worthwhile to them now and in later life? The Illinois Council of Teachers of Mathematics is trying to correct a situation. What are other states doing?

A Cross Number Puzzle for Valentine's Day



ACROSS

1. Date of Valentine's Day, February _____.
3. Valentine's Day is the _____th day of the year.
5. There are generally _____ days in the month of February.
6. There are _____ months in a year.
7. St. Valentine lived in the _____ century.
8. Seven months of the year have _____ days.
10. Number of letters in the month in which Valentine's Day is.
11. The number of days in a normal year multiplied by $3\frac{1}{4}$.
13. Four months of the year have _____ days.

DOWN

1. Two less than the product of $5 \times 5 \times 5$.
2. $8 \times 6 =$_____.
3. 246 divided by 6.
4. The product of 22 and 24.
8. In a normal year there are _____ days between Valentine's Day and Christmas. (Do not count Valentine's Day or Christmas Day.)
9. The number of quarters in $40.
11. The number of days in the year we send Valentines.
12. The number of days in a week plus one.

Contributed by MARGARET WILLERDING of San Diego State College, California

Developing Creative Thinking in Arithmetic

CALVIN H. REED
Lincoln, Nebraska

IN THIS DECADE OF 1950–60, we find ourselves faced with a paradoxical situation. On the one hand, we pay high tribute to the creative minds that have helped to make America great—politically, economically, scientifically, and socially. We bestow our admiration on our Einsteins, Carvers, Edisons, Salks, Jeffersons, and others, because they contributed much to make our lives richer and happier. These men were creative thinkers. On the other hand, as we review the contemporary scene, we can identify people, forces and institutions that are suppressing creative thought and opinions of individuals.

Standing as we are today on the threshold of great scientific, social, economic, and political developments, we can ill-afford to have an educational program aimed at uniformity of thought and action. Such education would be suicidal for America. We cannot expect our children today to solve their adult problems with our answers. Nothing less than an education firmly dedicated to developing creative minds will carry our country and the world through the years ahead.

Creativity not Limited to the Arts

It has been unfortunate that the terms creativity and creative thinking have been associated exclusively with the arts. To say that a person is creative or is doing creative thinking too often brings forth a mental image of the artist sweeping the brush across the canvas, or the musician creating a new rhythmical phrase, or the poet interpreting life in beautiful descriptive language, or the actor portraying human conflict. Seldom will the terms creativity or creative thinking bring forth a mental picture of a mathematician working on a new solution to a problem, or an elementary teacher devising new ways of making the abstract figure "eight" meaningful to children. This belief that creativity and creative thinking are in some way limited to the expressive arts has caused a false dichotomy within our elementary schools. In practice this has caused some teachers in departmentalized systems to believe that they develop creative thinking in growing children while the other teachers are concerned with the teaching of facts, understanding, and skills. Furthermore, it has resulted in teachers of self-contained classrooms to view certain aspects of the curriculum as contributing to creative thinking while the other content areas make little contribution.

Elements of Creative Thinking

A creative mind is a complicated mind. It is imaginative, full of hypothesis, theories and dreams. It questions all aspects of truth. It seeks to discover for itself the facts, generalizations, and understandings that may appear to be widely accepted. It involves inventiveness, productivity and the discovery or rediscovery of truth.

The first essential in creative thinking is a recognition by the individual of a disturbing element or a problem. This problem must be one that bothers the individual before he becomes energized to do something about it.

Along with this recognition of a disturbing element is the sensitivity to things which are irritants or problems. Some individuals are more sensitive to things that

bring dissatisfaction than others. It would seem that the teacher's job would be to guide the child to be alert and sensitive to dissatisfaction.

A second element in creative thinking is action on the part of the individual.

A third element, about which little seems to be known, is that of inspiration, or formulating hypotheses, or developing hunches or courses of action. The individual decides to do something to remove the dissatisfaction. He applies his previous learnings; he formulates possible solutions, and then tries them out to see if they will work.

It is obvious from this brief description of the essentials of creative thinking that the steps are similar to the scientific method of problem solving. The logical steps of problem solving given by Dewey are: (1) becoming aware of a problem, (2) clarifying the problem, (3) proposing hypothesis for solution of the problem, (4) reasoning out implications of hypotheses, and (5) testing the hypotheses against experience.

Creative Thinking in Arithmetic

Some of the essential principles a teacher should follow to develop creative thinking in arithmetic are:

1. *Problems become the mainspring to creative thinking.* Without a problem to solve the child has no motivation to think originally. Thus, the teacher's task is one of creating practical or intellectual problems which are meaningful to children. Furthermore, the problem must be one accepted by the child as his own in order for his energies to be directed into creative channels.

2. *The skillful teacher will guide the child in clarifying the issues of the problem.* The child should understand from the analysis what is required and what is known about the problem. This step seems to be an important one if the child is to perceive the problem in a clear focus and be ready to attack it.

3. *The formulating of hypotheses for the solution of the problem is where most of the creative thinking occurs.* It is during this stage of problem solving where the child formulates his hunches, relates his previous knowledge and skill to the problem at hand, tries out new relationships, and sets forth a possible solution.

Too often the teacher is impatient with the child for not giving a speedy answer. It should be remembered that thinking of a creative nature takes time and refuses to be rushed. Children differ greatly in their speed of response. The good teacher realizes this and makes provision in her arithmetic teaching for the slow and fast child.

4. *Knowing when to help and not to help a child when he is formulating hypotheses to a problem is of great importance.* It is my opinion that teachers of arithmetic are often afraid to give the child a chance to make a mistake. The teacher steps into the learning act at the first sign of difficulty and thereby deprives the child of an opportunity to do creative thinking. In many classrooms I have observed children waving their hands for help at the first moment difficulty is encountered. The teacher hurries about the room giving the solutions, and thus rescuing the child from the ordeal of thinking for himself.

On the other hand, too little help by the teacher can be equally destructive to creative thinking.

5. *Equally important to creative thinking are the questions the teacher asks the children.* During the arithmetic period the teacher should use questions, such as, What do we need to find out? How can we do that? How can we find how many? Now what do we need to do? What are we trying to find out? What shall we do? How did you discover that? What does this problem say? How did you know that was the answer? Is there another way of doing that? Can you estimate how many? Which do you think is the better answer? Is there another way of doing that? Can you prove your answer? Have we answered the questions?

6. *The key to creative thinking is the*

teacher herself and her attitude toward arithmetic. Unless the teacher is a creative person in her approach to problems and in developing a stimulating environment for quantitative thinking there will be few opportunities for children to think creatively.

Summary

Our elementary arithmetic program abounds with opportunities for children to do creative thinking. Children are creative. It is the teacher's responsibility to provide the means and satisfactions for children to use their creative abilities in learning the language of quantitative thinking.

The good creative teacher of arithmetic should be able to give an affirmative answer to these ten questions.

1. Do I help my children identify the arithmetical problems significant to them in terms of their experiential background of living?
2. Do I allow time for my children to take action in thinking through the solution of a problem before suggesting methods of solving it?
3. Do I through my questions and directions encourage my children to think for themselves?
4. Do I encourage my children to seek for solutions to a problem instead of *the* solution?
5. Do I show sincere appreciation and approval for a child's creative efforts although the answer was unsatisfactory?
6. Do I place as high a value on creative thinking as I do correct responses to drill problems?
7. Do I encourage my children to evaluate their own methods used in solving a problem?
8. Do I recognize the differences among my children in their ability to solve problems creatively?
9. Do I make a conscious effort to help my children understand that arithmetic is a quantitative way of thinking and that it offers opportunities for creative thinking?
10. Do I evidence creativity in my teaching methods, in the way I organize curriculum content, and in my own personal behaving?

EDITOR'S NOTE. Mr. Reed's argument for creativity in learning arithmetic is somewhat synonymous with the idea of *discovery* which has appeared a good deal in recent discussions of arithmetic. Thinking is the keynote. But children must be given both opportunity and time to think. The editor learned this many years ago when he asked a boy what he was doing (apparently nothing) and the boy responded, "I'm thinking." The teacher should set the stage for thinking and should know how to give little hints to stimulate thinking. Too often the inexperienced teacher, in her rush for answers, will do all the thinking for her pupils. It is a high art to be able to give just the right amount of help and encouragement to each pupil. Timing is also important. Too little and too late is as bad as too much and too often. Let us not forget that we need practice in thinking just as much as we do in skills. Particularly with young children we must remember that creative and inventive procedures are often very different from an adult's attack upon the same problem.

Illinois Council Meetings

In addition to a statewide meeting in October, the Illinois Council of Teachers of Mathematics will hold six sectional meetings this spring. At each of these a sectional meeting devoted to arithmetic in the elementary school will present interesting discussions under competent leadership. The sectional meetings as scheduled are: March 23—Granite City; March 30—Joliet; March 30—Carbondale; April 6—Normal; April 10—Charleston; and April 13—Kewanee.

Correction—Benz, Division

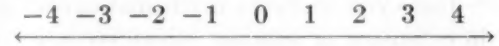
In the November issue on page 187, column one, about two thirds of the way down, an error in typesetting occurs. The example should have been 42)936 instead of 42)369. This is regretted because it confuses the argument in the text.

Teaching the Digit Zero

CARL N. SHUSTER
Trenton, New Jersey

THE LITERATURE OF ARITHMETIC is filled with such absurd expressions as "*zero is only a place holder.*" As a result thousands of arithmetic teachers are teaching this nonsense and doing a vast amount of harm.

Zero has a very respectable place on the real number line.



If 0 is merely a place holder on the number line, then the same is true of −3, +4, +240 etc. This is clearly absurd.

In the number 603 the digit zero is a place holder and in addition indicates that there are *no* tens. In the number 693 the digit 9 is a place holder and indicates that there are *nine* tens. Since the digits 0 and 9 in the above numbers are both significant and have identical roles (acting as place holders and indicating the number of 10's) it would be inane to say that the zero was *only a place holder.*

In numbers 603, 6003, 60003 etc. the zeros are significant and have the double roles that the nines in 693, 6993 and 69993 have.

In the measurements 9 ft. .0 in., 90/128 in., and 2.000 in. the zeros are significant digits and certainly are not *merely place holders.* The measurements 9 ft. .0 in., 90/128 in. and 2.000 in. all have four significant digits. They differ greatly in both accuracy and precision from the one significant digit measurements: 9 ft., 9 in., and 2 in.

In high school and college, the pupil will learn that $a°$ or $2°$ or $(-375)°$ are all equal to 1. He will learn that the value of one of the unknowns in an equation can be 0. He will learn that it is cold when the temperature is 0°. When he wishes to find the cologarithm of a number he will use the fact that $\log 1 = 0$. When he balances a set of books he may find that \$638.46 − \$638.46 = 0. He will find that if $r = 0$ there is no correlation. He will learn that the average of +4°, +6°, and −10° is 0°. He will learn that if the discriminant of a quadratic equation is 0 the roots are real, rational and equal. He will learn that if $dy/dx = 0$ the function he is working with will have a maximum or minimum point. He will learn that,

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1,$$

etc. etc. Certainly the zeros in the above expression are not *merely place holders.*

Zero does, however, have a unique function. It is the only one of the ten digits that can appear in a correctly written answer without being significant.

In the measurement's 600.00 ft. and 1.00020 in. all the digits are significant. However, if the equatorial radius of the earth, 3963.399 miles is rounded to 4000 miles, the three zeros will not be significant. Again, if the two digit measurement 2.43 mm is changed to 0.00000243 Km, the zeros will not be significant.

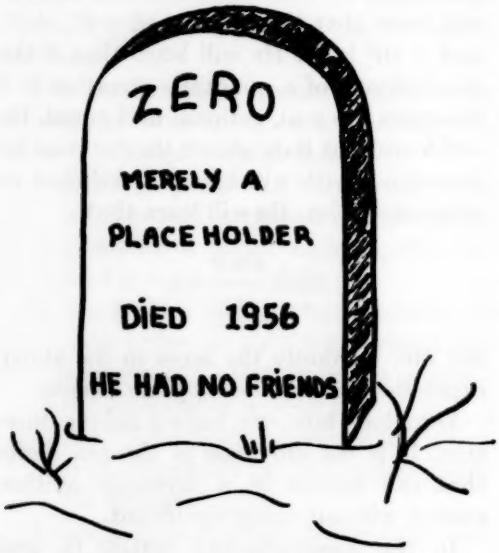
Pupils are often taught to "round" numbers in the fifth grade. They should be taught the meaning of significant digits when they are taught to round numbers. They should learn that zeros used to replace the digits dropped in rounding numbers are not significant.

It took the human race many thousands of years to invent zero and our decimal place value number system. It took a thousand years for this vastly better

number system to displace Roman numerals, and even now we are prevented by mental inertia from cashing in on the full value of the decimal number system.

We cannot expect children to develop a full concept of zero in any one grade. Like most other concepts the concept of zero must grow or develop with the child. We can, however, abstain from teaching false concepts that are very difficult to eliminate.

Let us hope that the phrase *zero is merely a place holder* will soon become extinct.



EDITOR'S NOTE: Poor little round fat zero! Dr. Shuster has buried him with a marker to mark his role as a "place holder." But Shuster has shown us that zero is much more, in fact he is a rather sophisticated fellow. And there is probably the cause of most of the trouble. It is the teacher who lacks Mr. Shuster's background in mathematics who relegates zero to the one simple role. At one stage in a child's development, zero may be predominantly a place holder but at this same time the "zero value" concept is also very real. Then as a child progresses he must certainly learn other roles for zero. Yes, zero is rather sophisticated and let us, as teachers, not reveal our lack of sophistication in mathematics by dismissing zero merely as a place holder.

Do You Like Arithmetic?

Have you seen one of the newer colorful mathematics books for children?

How recently have you read "Alice in Wonderland and Through the Looking Glass" or "Gulliver's Travels"?

Have your pupils brought you a problem of their very own to solve in class?

Have the new articles concerning the computing machines and games with other number bases than ten come to your attention?

Have you read about field trips and bulletin boards and the wonders of 2 in the publications of the National Council of Teachers of Mathematics?

Have you studied a mathematics course in extension or summer school recently?

Have you visited other teachers of mathematics in classes and grades different from yours?

Have you seen mathematics in nature perhaps from the window of a train or an airplane or in a tiny shell or seed pod brushed aside by a passing foot?

Have you visited a new industry or a science museum to see mathematics at work?

Have you thought of the basic position that mathematical thought plays in developing our modern civilization?

Have you made with your own hands a device to demonstrate a mathematical idea?

Have you watched a creator of mathematics at work?

Have you felt the excitement of discovery in solving a problem that is new, at least to you?

Does the spirit and enthusiasm you feel spill over into your teaching?

Do you like arithmetic?

Contributed by DOROTHY M. SWAN

A Bibliography of Mathematics Books for Elementary School Libraries

ADRIEN L. HESS
Montana State College, Bozeman

SOME RECENT TEXTBOOKS on the teaching of arithmetic have indicated the existence of a need for a bibliography of mathematics books for elementary school libraries. Spitzer[1] states that "there are many stories in which counting plays an integral part which may be read to children of pre-school and school age." He lists ten books which would be useful for this purpose. Wilson[2] devotes a chapter to "Appreciation Units in Mathematics" in which he discusses some books which would be helpful in developing appreciation for mathematics.

Hutcheson, Mantor, and Holmberg[3] recently published an extensive bibliography of books suitable for use in elementary school libraries. Since this bibliography was prepared a number of books have appeared which should be included in such a list.

The annotated bibliography included below, based on an examination of each book included, will acquaint elementary school teachers and librarians with as many of these new books as are available to the writer, as well as a few older books.

Lower Grades

	Grade	Price
BANIGAN, SHARON. ONE BY ONE. Author illustrated. np. 1953. Hampton. Simple facts of addition are given in rhyme and in picture. A row of beads on a wire encourages the youngster to carry out these simple additions. Durable cardboard pages.	1, 2	$1.00
BRODY, VIRGINIA. 'ROUND THE CLOCK BOOK. Pictures by George Kanelous. Harvey House, np. 1956. The face of a clock with moveable hands which can be set as each page of verse is read enables children to learn to tell time. The highly colored pictures depict familiar scenes to accompany the hours of the day. Heavy cardboard makes the book quite indestructible.	K, 1, 2	1.00
BUDNEY, BLOSSOM. A KISS IS ROUND. Illustrated by Vladimir Babri. np. 1954. Lothrop. In this modernistic picture book the concept of round is vividly developed by many familiar illustrations in rhyming verse and brightly colored pictures. The concept of square is introduced to contrast with the concept of round.	K, 1, 2	2.50

[1] Spitzer, Herbert F.: *The Teaching of Arithmetic*, Second Edition, Houghton, Mifflin Co., New York. 1954. 416 pp. (pp. 26–27).

[2] Wilson, Guy M. and others: *Teaching the New Arithmetic*, McGraw-Hill, New York. 1951. xiv+483 pp. (pp. 320–349).

[3] Hutcheson, Ruth; Mantor, Edna and Holmberg, Marjorie. The Elementary School Mathematics Library—A Selected Bibliography, THE ARITHMETIC TEACHER. Feb. 1956. 3: 8–16.

	Grade	Price
DUVOISIN, ROGER ANTOINE. TWO LONELY DUCKS. Author illustrated. 36 pp. 1955. Knopf	K, 1	2.00
In this entertaining story the numbers from one through ten appear in sequence in large numerals and in words. A sequence of first, second, third, etc., as well as counting the days of the week and the weeks in a year are a part of the story. The drawings make this more than just a counting book.		
HALL, WILLIAM. TELLTIME THE RABBIT. Pictures in three colors by Charlotte Steiner. 28 pp. 1955. Crowell	K, 1	2.00
This book is a reissue, in a completely new format, of an old favorite. The gay pictures and the easy words make it a good book to read aloud at story telling time. The amusing story makes learning to tell time easy and fun as well.		
HALL, WILLIAM AND ROBIN. TELLTIME GOES A'COUNTING. Pictures by Charlotte Steiner. np. 1956. Crowell	K, 1	2.00
A sequel to Telltime the Rabbit. Telltime learns to count from 1 through 10 by counting his animal friends: two colts, three squirrels, and so on through ten mice. Each group of animals is pictured in colors, as well as the one hundred animals invited to Telltime's party.		
KAULA, EDNA M. ONE, TWO, BUCKLE MY SHOE. Pictures in color. np. 1951. Whitman	K	.15
A counting book which uses the old favorite "1-2 Buckle My Shoe, 3-4 Knock On The Door, etc." Red colored numbers from 1 through 30 are on colored pictures. The pictures illustrate the words accompanying each picture, not the number on the page.		
KING, DOROTHY N. SET THE CLOCK. Pictures in full color. np. 1946. Watts	K	1.00
The youngster may set the plastic hands on a cardboard clock to agree with the time of each of a series of events in Joan's day from the time she arises in the morning until she goes to bed in in the evening. In rhyme.		
MEEKS, ESTHER K. ONE IS THE ENGINE. Illustrated by Ernie King. 24 pp. 1947. Follett	1, 2, 3	1.25
This is a train book, a counting book and a story book. Any child who loves trains will learn to count while identifying the different cars.		
MOORE, LILIAN. MY FIRST COUNTING BOOK. Pictures by Garth Williams. 22 pp. 1956. Simon and Schuster. A Golden Book	K	1.00
The book is printed on heavy cardboard. The animal pictures are very realistic. The rhymes are about one puppy, two woolly lambs, and so on up to ten nuts for a little squirrel.		
REED, MARY AND OSSWALD, EDITH. THE GOLDEN PICTURE BOOK OF NUMBERS: WHAT THEY LOOK LIKE AND WHAT THEY DO. Illustrated by Corrine Malvern. 80 pp. 1954. Simon and Schuster	1, 2, 3	1.00
Numbers are woven into all the easy-to-read stories and color-		

	Grade	Price
ful pictures to bring out their meanings in everyday life. These uses include time, simple combinations of numbers, and concepts of money, big, small, short, tall, etc. This book should prove to be very popular with children.		
SCHLEIN, MIRIAM. IT'S ABOUT TIME. Illustrated by Leonard Kessler. 48 pp. 1955. Scott	1, 2, 3	2.00
Verse and haphazard-looking pictures are used to develop the concept of time. The various units of time, such as second, minute and hour, are introduced as well as the telling of time.		
SCHLEIN, MIRIAM. SHAPES. Pictures by Sam Berman. 40 pp. 1952. Scott	K, 1, 2	2.00
A book of familiar things written to help the small child develop an understanding of the concept of shapes. Young children will enjoy the many illustrations.		
THOMAS, JOAN GALE. ONE LITTLE BABY. Illustrated by author. Pictures and verse. np. 1956. Lothrop	K, 1, 2	1.00
Numbers from one through ten are associated with familiar figures appearing in the well-known Christmas Story. The simple verses in this Christmas counting book are quite appealing.		
TRUE, LOUISE AND OWENS, LILLIAN. NUMBER MEN. Pictures by Lillian Owens. 32 pp. 1948. Children's Press	1, 2	2.30
The directions for writing the numbers from one to ten are given in verse. The four-color pictures teach the children to count.		
TUDOR, TASHA. 1 IS ONE. Illustrated by author. np. 1956. Oxford	K, 1	2.75
A counting book charmingly illustrated by a well-known artist in soft colors and in black and white. Each number from one through twenty is accompanied by an original verse. Pictures of animals, children, flowers and trees make concrete the numbers presented. The vocabulary and the peaceful scenes seem more typically English than American.		
WATSON, NANCY DINGMAN. WHAT IS ONE? Illustrated by Aldren A. Watson. 42 pp. 1954. Knopf	K, 1	2.00
An older brother uses quite simple and commonplace examples to teach his younger sister the meaning of the numbers from one to ten. The drawn pictures make the examples very real istic.		
WATSON, NANCY DINGMAN. WHEN IS TOMORROW? Color illustrated by Aldren A. Watson. np. 1955. Knopf	K, 1	2.00
Charming pictures and a simple story give youngest readers an understandable concept of time.		
ZINER, FEENIE AND THOMPSON, ELIZABETH. THE TRUE BOOK OF TIME. Illustrated by Katherine Evans. np. 1956. Children's Press	1, 2, 3	2.00
Simple text and outline drawings decorated with a single color are used to describe a number of ways that have been used to measure time. Some practice in telling time may be gotten from the two hands and face of a clock included in a number of pictures. This book should lead to an understanding of the measurement of time.		

	Grade	Price
ZOLOTOW, CHARLOTTE (SHAPIRO). ONE STEP, TWO .... Illustrated by Roger Duvoisin. np. 1955. Lothrop. Ellen takes a walk in which the steps are counted from one to twelve. The many illustrations and the large print may encourage beginning readers to take over the story hour for the younger children.	K	2.00
———. 1, 2, 3, 4, 5. Photographs by Robert Doisneau. Verses by Arthur Gregor. np. 1956. Lippincott. Superb photography has been used to display concrete objects for the numbers 1 through 12. Although some of the photographs are unmistakably French, the objects portrayed will attract the attention of all children. Rhyming verses accompany each number. A handsome book.	K, 1, 2	2.50
———. I LEARN MY NUMBERS. Illustrated by Jim Short. np. 1955. Gabriel. The numbers and pictures are quite large for easy reading. The brightly colored pictures and objects depicting the numbers from one through ten will hold the interest of children. The addition facts of 1+1, 2+1, and 2+2 are cleverly introduced.	K	1.77
———. THE TRUE-TO-LIFE ABC BOOK, INCLUDING NUMBERS. Illustrated by Johan Polak. np. 1952. Grosset. Two pages of the book are devoted to pictures of commonly known animals which a youngster can count. Format may be large for a small child.	K	1.00

Upper Grades

ADLER, IRVING. TIME IN YOUR LIFE. Illustrated by Ruth Adler. 127 pp. 1955. Day. Many fields of knowledge are considered which need time as a basis for understanding. Among these are: man's first timepiece; the stars and planets; rocks and rivers; history and the calendar; time zones; music and dancing and others. The illustrations aid the reader in understanding the text. Directions for a perpetual calendar are included. The interestingly written book can be enjoyed by young and old.	5 up	2.75
ANDERSON, RAYMOND. ROMPING THROUGH MATHEMATICS. Drawings, charts, diagrams by Harry Zachary. 152 pp. 1947. Knopf. Explanatory drawings, diagrams and charts are used to make clear and interesting topics of arithmetic, algebra, geometry and calculus. Might inspire better students to make a more serious study of mathematics.	7 up	3.00
FRIEND, NEWTON. NUMBERS: FUN AND FACT. Illustrated. 208 pp. 1954. Scribner. A miscellany of mathematical oddities, curiosities, puzzles and and problems. Readers learn how the number system and various mathematical concepts came into being. Useful for enriching elementary mathematics.	7 up	2.75

	Grade	Price
GILLES, WILLIAM F. THE MAGIC AND ODDITIES OF NUMBERS. 65 pp. 1953. Vantage	7 up	2.75

The mathematical oddities and stunts included in this volume will provide enrichment materials for upper grade arithmetic. The mystery and superstition associated with the numbers seven and thirteen will arouse the curiosity of students in junior high.

HOGBEN, LANCELOT. THE WONDERFUL WORLD OF MATHEMATICS. Illustrated. 69 pp. 1955. Garden City	5 up	2.95

The growth and development of mathematics through the ages is told in story form and is portrayed by pictures. Lavish illustrations depict this development so colorfully and yet so simply that young and old alike will be fascinated.

KOJIMA, TAKASHI. THE JAPANESE ABACUS. Illustrated. 102 pp. 1954. Tuttle	7 up	2.00

A brief history of the abacus and the basic principles of calculation are given. The four fundamental arithmetic operations are demonstrated with many examples and diagrams. This book will enable any interested person to learn to use the abacus.

SAWYER, W. W. AND SRAWLEY, L. G. DESIGNING AND MAKING. Illustrated by Srawley. 192 pp. 1952. Oxford	7 up	2.75

Numerous mathematical and scientific principles are used for many interesting experiments and problems. These experiments and problems are designed to lead the pupil on a genuine search for knowledge. Because of the lack of mathematical knowledge progress may be held up at some stage of the experiment or problem.

SHACKLES, G. L. S. MATHEMATICS AT THE FIRESIDE. 165 pp. 1952. University Press	7 up	3.25

A conversational approach is used in this book to develop number concepts and some basic ideas from algebra and geometry. The children in the book may be too young for the older children who will be capable of reading it.

WEEKS, RAYMOND. BOYS' OWN ARITHMETIC. Illustrated by Usabal. 188 pp. 1924. Dutton	6 up	2.50

Profusely illustrated to match the author's spirit, this humorous book introduces many words unknown to the reader in the more than 100 stories, each of which contains an arithmetical problem. Considerable mental exertion must be expended to obtain the solutions, which are often not nice even numbers. Some of the data is out-of-date. The problems when read aloud will capture the interest of many listeners.

EDITOR'S NOTE. This bibliography supplements the one published in THE ARITHMETIC TEACHER in February, 1956. Copies of the former bibliography are available from the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington 6, D. C. at 20 cents each. Ask for "The Elementary School Mathematics Library."

A directory of names and addresses of publishers of the items in the current bibliography is printed on the next page.

Directory of Publishers

CHILDREN'S PRESS. Children's Press, Jackson Blvd. and Racine Ave., Chicago 7, Ill.
CROWELL. The Thomas Y. Crowell Company, 432 Fourth Avenue, New York 16, N. Y.
DAY, John Day Company, Inc., 210 Madison Avenue, New York 16, N. Y.
DUTTON, E. P. Dutton and Co., Inc., 300 Fourth Avenue, New York 10, N. Y.
FOLLETT. Follett Publishing Company, 1010 West Washington Blvd., Chicago 7, Ill.
GABRIEL. Sam'l Gabriel Sons and Co., 200 Fifth Avenue, New York 10, N. Y.
GARDEN CITY. Garden City Books, Garden City, N. Y.
GROSSET. Grosset and Dunlap, 1107 Broadway, New York 10, N. Y.
HAMPTON. Hampton Publishing Company, Inc., 460 Fourth Avenue, New York 16, N. Y.
HARVEY HOUSE. Harvey House Publishers, Irvington-on-Hudson, N. Y.
KNOPF. Alfred A. Knopf, Inc., 501 Madison Avenue, New York 22, N. Y.
LIPPINCOTT. J. B. Lippincott Co., E. Washington Square, Philadelphia 5, Pa.
LOTHROP. Lothrop, Lee and Shepard Company, Inc., 419 Fourth Avenue, New York 16, N. Y.
OXFORD. Oxford University Press, Inc., 114 Fifth Avenue, New York 11, N. Y.
SCRIBNER. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, N. Y.
SCOTT. William R. Scott, Inc., 8 West 13th St., New York 11, N. Y.
SIMON AND SCHUSTER. Simon and Schuster, Inc., 630 Fifth Avenue, New York 20, N. Y.
TUTTLE, Charles E. Tuttle Company, Rutland, Vt.
UNIVERSITY PRESS. University of Chicago Press, 5750 S. Ellis Avenue, Chicago 37, Ill.
VANTAGE. Vantage Press, Inc., 120 West 31st St., New York 1, N. Y.
WATTS. Franklin Watts, Inc., 699 Madison Avenue, New York 21, N. Y.
WHITMAN. Whitman Publishing Co., Racine, Wis.

Test Review

New York Test of Arithmetical Meanings, Levels One and Two by J. Wayne Wrightstone, Joseph Justman, Morris Pincus, Ruth H. Lowe, Yonkers-on-Hudson, New York and Chicago, Illinois, World Book Company, 1956.

These tests measure mastery of pre-measurement and numerical concepts. *Level One* is for use during the last half of Grade 1 and the beginning of Grade 2, while *Level Two* is for use during the last half of Grade 2 and the beginning of Grade 3. The test items at both levels are pictorial and there is no overlapping between the two. The authors feel the test can serve both as a measure of achievement and readiness to go ahead in the arithmetic program.

The test items seem valid on the basis of materials and courses of study used in these grades. Each item has been approved by teachers of the included grade levels.

The administration of the test takes one hour. The test is divided into two parts, Pre-measurement concepts and Mathematical concepts, each taking thirty minutes. At least one hour should be allowed between parts. The test manual is complete and easy to follow. In the writer's experience, the youngsters had no trouble following directions and seemed to know what to do.

Percentile norms are provided in the manual as well as a five-category rating system. In the writer's opinion, the rating system seems much more appropriate for youngsters in the early grades. These norms were drawn from a sample of approximately 17,000 students. Included were 62 school systems from 28 states.

Split-half reliability coefficients vary from approximately .80 to .90 after correction by the Spearman-Brown formula. Reliability coefficients before corrections are not published. The related data also includes an approximate standard error, which the authors wisely point out should be considered in the interpretation of any individual or class score.

For further information and a sample of test items refer to Dr. Wrightstone's description of the test in April, 1956 issue of THE ARITHMETIC TEACHER.

ROBERT VAN DAM

Meanings in Division

H. C. Christofferson
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Whenever one word has several meanings there is danger of misunderstanding. The boy who thought slide rule was spelled "sly drool," could be expected to have some difficulty.

When stationed at Florence, Italy, after the war, the writer and a friend approached an artist who was painting portraits. I said, "Do you mind if we watch you?" The artist looked puzzled, then pointed to her head and said, "mind," then to her wrist watch and said "watch." Finally said, "Non capisco," which means, "I do not understand." Yet she knew the meaning of both mind and watch.

Then there is the classic retort of a third grade boy when asked, "How many feet in a yard?" He replied, "You can't tell until you know how many children are playing there."

While the elementary teacher uses "weasel words," words with different meaning, less in arithmetic than in almost any other subjects, yet there are basic difficulties. Subtraction is often taught as synonymous with "take away," yet it has other meanings such as *difference*, *how much more*, and *separation* into component parts, which are just as important. These meanings and those of addition have been emphasized so much in recent books, and in articles in The Arithmetic Teacher, that this discussion will be limited to division, possibly to be followed by one on multiplication.

Recent treatments of division describe the two meanings, measurement and partition, as though this were the whole story. There are at least two other very important meanings of the verb to divide, as used in arithmetic, as well as a dozen related non-mathematical meanings. Let us first briefly state the meanings of division as (1) "measurement" and as (2) "partition." This can be most readily done by the use of concrete data.

Division as "Measurement"

The exercise, 12 yd.$\div$2 yd., means to ask the question, "How many times can 2 yd. be subtracted from 12 yd. or how many times can 2 yd. be *measured* off on 12 yd.?" The answer is the abstract number 6. Note that divisor and dividend must be expressed in like units and that the quotient is always abstract. 12 yd.$\div$2 ft. would not be 6. The 12 yds. should be changed to feet or the 2 ft. to yd. to divide. Even though all the numbers are abstract; that is, $12 \div 2 = 6$, the thinking is still how many 2's are there in 12 ones, or how many times can 2 of anything be measured off on 12 of the same quantities.

One interpretation for this problem would be to consider all computation as being done with abstract numbers, and that denominations are attached which make sense. For example, 2 ft.+3 ft.=5 ft. Clearly only the 2 and 3 are added, not the feet. 2×3 ft.=6 ft. Again $2 \times 3 = 6$, so 2×3 ft.=6 ft., only the abstract numbers are multiplied. To the author of this article this seems mere semantics and is not helpful in teaching children. Scientists also prefer keeping the units with the factors to avoid errors. In this presentation some factors will be considered as concrete quantities and others as abstract numbers in order to emphasize certain problems of understanding meanings.

Division as "Partition"

Using concrete data again, 12 yd. ÷ 2 = 6 yd., this means that if 12 yd. is divided into 2 equal parts that each part has 6 yd. in it, hence the word *partition*. Note that now the divisor indicates the number of pieces or parts. It must be an abstract number and an integer. Perhaps one could give meaning to 12 yd. ÷ $1\frac{1}{2}$ = 8 yd. and show that $1\frac{1}{2}$ parts each 8 yds. would mean 1 part, 8 yd. long; and 1 part, $\frac{1}{2}$ of 8 yd. long; but this would be puzzling and unreal. In partition, unadulterated or unsoftened, the divisor must be an abstract integer and the dividend and quotient must have the same denomination. Even though all three terms are abstract the divisor must be an integer. Thus 12 ÷ 2 = 6 can mean two different relationships: (1) how many 2's can be subtracted from 12 or *measured* off on 12, and (2) if 12 is divided into 2 equal parts, there will be 6 in each *part*.

Division as "Ratio"

Not until one begins to teach percentage does one have a great need for two other basic meanings of division. Basically the so-called "2nd and 3rd case" in percentage can be solved by the "product-divided-by-factor" analysis. Whatever the analysis the division needed is the same. Here are two important problems from which these mathematical difficulties emerge.

(1) What per cent income do I get on my money if I invest it in A.T.&T. stock at $180 per share and it pays $9 per year in dividends? That is, what per cent of 180 equals 9?

(2) At what price would I buy A.T.&T. stock which pays $9 per year in dividends, if I wish to realize 6% interest on my money? That is, 6% of what price = $9?

These problems can be solved by applying a fundamental principle, namely, *if 2 numbers are multiplied together to obtain a product, then the product divided by either factor equals the other factor*. For example, if 2 × 3 = 6, then 6 ÷ 3 = 2 or 6 ÷ 2 = 3. Also, if $\frac{1}{2} \times 20 = 10$, then $10 \div 20 = \frac{1}{2}$ and $10 \div \frac{1}{2}$ must equal 20. Division must have such meanings as to make these solutions possible. Suppose we analyze the first one from the investment problem. Here one divides $9 by $180 to get .05 or 5%. $9 ÷ $180 = ?

Is this measurement? The divisor and dividend have the same units of measure like 12 yd. ÷ 2 yd. However, one does not determine how many shares each costing $180 can be subtracted from or measured off on the $9 dividend. He is concerned, however, with the *ratio*,

$$\$9 \div \$180 = \$9/\$180$$
$$= 1/20 = 5/100 = .05 = 5\%.$$

Whenever a small number is divided by a larger number, the meaning is not measurement, but ratio. Sometimes when a large number is divided by a small one the meaning may also be ratio. For example: a football player weighs 210 lbs. and a tennis player weighs 140 lbs. Divide: 210 ÷ 140 = $1\frac{1}{2}$. This tells that one man weighs $1\frac{1}{2}$ times as much as the other, or if I divide the other way, then one weighs $\frac{2}{3}$ as much as the other. Division often means ratio.

A Fourth Meaning of Division, You Name It

To solve the second percentage problem, 6% of ? = $9, one divides, $9 ÷ .06 = $150, the cost of one share to yield 6% interest. Clearly this is not measurement nor ratio because the divisor is abstract. It is not partition, because one does not divide $9 into .06 parts and get $150 in each part, even though the divisor is abstract. Yet the application of the general law, that *the product of 2 factors divided by one factor equals the other factor*, will frequently result in this situation and of course it frequently occurs in life situations.

A dress sells for $24 and this is at a discount of $\frac{1}{4}$ from the original price. That is, $\frac{3}{4}$ of the original price = $24. Then $24

$\div \frac{3}{4} = \$24 \times \frac{4}{3} = \$32$, the original price. Check: \$32 less $\frac{1}{4}$ of \$32 = \$24.

Whenever one wishes to find the whole when a part is known, or to find the base when the per cent or ratio is known, then one may divide a concrete quantity by an abstract ratio and get as an answer a larger or smaller concrete quantity, depending upon the ratio being less than unity or greater than unity.

Notice also that this meaning is to be distinguished from any division by a fraction: (1) 12 yd. $\div \frac{2}{3}$ yd. = 18, and (2) 12 yd. $\div \frac{2}{3} = 18$ yd. In (1) the meaning is clearly measurement, how many pieces $\frac{2}{3}$ yd. each can be cut from 12 yd.? But 12 yd. $\div \frac{2}{3} = 18$ yd. can only mean $\frac{2}{3}$ of ? = 12 yd. It is the reverse of multiplication when the abstract factor is a ratio; or, it is finding the whole when a part of it is known. It is not partition in the same sense that partition means dividing a concrete quantity into a number of parts.

Other Division Situations

(1) If I drive 200 miles in 5 hrs. at what rate do I drive? Answer: 40 mph, not 40 miles. In other words the ratio,

$$\frac{200 \text{ mi.}}{5 \text{ hr.}} = \frac{40 \text{ mi.}}{1 \text{ hr.}} = 40 \text{ miles per hour.}$$

Here the ratio is between unlike quantities, but both concrete.

(2) Starting from scratch I attain a speed of 40 mph in 4 minutes. What is my acceleration?

$$\frac{40 \text{ mph}}{4 \text{ min.}} = \frac{10 \text{ mph}}{1 \text{ min.}} = 10 \text{ mph per min.,}$$

my acceleration.

(3) In irrigation a farmer is entitled to 100 acre-feet of water. How deep will this be on 200 A.?

$$\frac{100 \text{ acre ft.}}{200 \text{ acres}} = \frac{\frac{1}{2} \text{ ft.}}{1} = .5 \text{ ft.}$$

Or the depth all over can be 6 inches.

(4) $12\frac{1}{2}$ ft. $\div 2\frac{1}{2}$ ft. = 5, but $12\frac{1}{2}$ ft. $\div 2\frac{1}{2}$ must equal 5 ft.

Where could this ever occur? Many solutions in surveying, in which one distance is multiplied or divided by an abstract ratio in the form of a tangent, sine, or cosine, involve this fourth meaning of division.

Basic Question

Is mathematics a living science which can adjust its meanings, its definitions, its techniques to the needs and varying situations found in life? Or is it an arm-chair, ivory-tower perfectionism that cannot change? The answer for most teachers must be the former. If this is the case then division cannot be defined completely by the two concepts of measurement and partition. Ratio must be added, and also a concept of reversed multiplication in which the divisor is an abstract ratio, you give it a short name.

EDITOR'S NOTE: Division is an arithmetical process. Professor Christofferson has discussed the meanings and significance of the process when applied to situations in which division is useful in obtaining an answer. He has pointed out that "measurement" and "partition" do not cover all the uses of division. Of course this depends somewhat upon how one defines these terms. For example, it is assumed that measurement must yield an integral answer. But let us consider: 4 cu. ft. $\div$ 8 cu. ft. in the case of having on hand 4 cu. ft. and a formula calls for 8 cu. ft. This can then be considered to yield $\frac{1}{2}$ batch. Or must this be thought of as ratio?

In teaching, should we try to have children understand all of the uses of division and their relationship to the process and to have these clearly defined in the minds of pupils? Case III percentage is commonly treated by simple algebraic analysis and solved in equation form. Must we then give this a special name or meaning in division? How do most children learn to sense that a situation calls for the division process? It is by separating "measurement" and "partition" or is it better merely to learn division and the uses thereof without technical differentiation? Who will carry the argument further?

Teaching the Three A's in Elementary Mathematics*

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RECENTLY I HAD OCCASION to think somewhat about the teaching of elementary arithmetic, algebra, and analysis —the three A's of mathematics. It occurred to me that some mathematics teachers might be interested in the personal reactions of a mathematical physicist, although these may well be commonplace to education experts. Let me begin with the first A—arithmetic.

C. F. Gauss boasted that "Mathematics is queen of the sciences and arithmetic the queen of mathematics." Earlier when J. Lagrange had been asked by the French tribunal what useful thing he could do to deserve life, he replied, "I will teach arithmetic." The Greeks themselves differentiated between arithmetic per se, which was essentially theoretical, and logistics, which was primarily computational. Even nowadays principles of procedures and applications to problems are both important for arithmetic teaching, just as understanding of grammar and use of a dictionary both contribute to language teaching. Let us look, first of all, at the natural numbers, which are the foundation of modern mathematics in contrast with the basis of Greek mathematics, which was geometry with its points and lines. L. Kronecker once claimed that "God created the natural numbers; every thing else is man's handiwork."

Children should begin and end with concrete experiences, not with abstract concepts, which must be developed gradually. For example, the very concept of unity is far from obvious in the relation between *one* banana and *one* bunch of bananas. What is common to 2 pennies, to 2 pigs, to 2 pencils—not to mention a brace of pheasants and a couple of people —is a matter of major abstraction. The perceptive limit itself of a number of objects is an interesting psychological question for different individuals. What, moreover, would be the result of adding one cat and one dog (I do not have in mind one fight)? It seems to me that this question has a legitimate answer, viz., two animals, inasmuch as we proceed thus from individual species to a broader genus. Most general, of course, will be a relationship like $1+1=2$, which applies to every thing.

It is important also to realize historically that the Greeks had no use for a number like zero, which is nowadays an integer. (They actually avoided the void.) To be sure, zero may mean nothing, but then again it may signify something. In a number like 20, the cipher (empty) means no units by virtue of its position, just as the number 2 has different significances in 2, 20, 200 (consider also, the additive positional notation for Roman numerals, e.g., CXVIII, which represents $100+10+5+3$). In this connection the teacher himself should be mindful of the fact that $1+1=10$ in the binary system (base 2) of units (cf. the decimal system with the base 10, the base 20 of the English score, the French vingt, the Aztec day (20 hours), and the base 60 for Babylonian degrees, minutes, seconds, et al.). The binary multiplication table, moreover, is very simple, viz.,

* From a talk given to the faculty of the Beauvoir Elementary School, Washington, D.C. and to the Mathematics Section of the Maryland State Teachers Association at Baltimore.

Multiplication Table
(Binary)

	0	1
0	0	0
1	0	1

Hence $10\times10=100$, i.e., $2\times2=4$ in our decimal notation. It is all important that the youngster be introduced early to the idea of positional notations so as to appreciate the economic value of using a small set of digit symbols to represent all numbers. In view of the abstract nature of number let us not take *any number* of things for granted.

The very idea of counting, moreover, is not always obvious. For many children the first acquaintance with numbers may be no more than a memory feat like "eeny, meany, miney, mo" or "one, two: buckle your shoe," etc.—the question "which" precedes the query "how many." We adults, too, are often accustomed to using numbers merely for names as in the case of a license number, or a telephone number, or a street address, or even a convict's designation. For example, even for adults the hour of the day more frequently signifies a time sequence rather than an actual duration. Strictly speaking, one should start with the idea of a cardinal number (from the root word meaning "hinge" or "turning point" and signifying chief in importance), which represents magnitude, i.e., how far, how long, how much, how many. Here to develop familiarity with the meaning of less than, of greater than, and of equal to one should encourage matching between groups of objects (cf. digits (fingers), pebbles (calculate), notches (tally), strokes (Roman 1–10, Aztecs 1–20), a game like musical chairs, et al.) involving what mathematicians call one-to-one correspondence. Ordinal numbers, which signify an ordering, may then logically be identified with the cardinal classes (according to magnitude). Counting off by pairs, by fives, et al., is a good means of relating ordinals and cardinals (cf. the number of dimes equivalent to a given group of pennies, the number of quarters equivalent to a given pile of nickels, etc.). Above all, experiential relations to things is a psychological prerequisite for understanding counting. Thus the abacus (cf. the importance of position on the board) may still be a useful tool for teaching arithmetic and, like fingers, should not be discarded for this purpose until no longer actually needed. Although we retain the names square and cube in our mathematical vocabulary we are inclined to forget that they originated in describing a square array of things and a cube array of things, respectively.

As a physicist, I would urge early correlation of counting with simple measurements, which are part of our daily lives, e.g. scales, thermometers, et al. Nowadays it is strange to find elementary school teachers avoiding the use of a simple rule like a yard stick, but at the same time encouraging youngsters to tell time with a much more complicated circular scale, viz. a clock, which has two pointers moving at different rates for different units. In measurement, moreover, the youngsters learn from experience that answers rarely come out even. He finds natural subdivisions convenient. He learns to guess inexactly first and then to seek exact answers—to think qualitatively before working quantitatively. For example, a student should recognize that 21×32 is approximately 20×30 (600) before developing the technique of precise multiplication (672).

It is quite important that a teacher of arithmetic have some comprehension of the modern theory of numbers itself. He should know the five basic laws, viz., the laws of commutation for addition and for multiplication, the laws of association for addition and for multiplication, and the law of distribution for multiplication. In specifying different rules in each of these cases one obtains different kinds of arith-

metic. A simple illustration of the practical importance of commutation is afforded by the mixing of chemicals: for example, the addition of sulfuric acid to water is quite harmless, but the addition of water to sulfuric acid is very dangerous. The student, of course, will readily appreciate the importance of commutation for division, i.e., 3 divided by 2 is not equivalent to 2 divided by 3. It would be instructive, I believe, for young people to learn these basic laws of commutation, association, and distribution with respect to numbers rather than to have to await their use later in algebra.

A child will certainly find it experientially interesting that two rows of three dots are equivalent to three rows of two dots, i.e., $2\times3=3\times2$.

He will thus "discover" arithmetic for himself. Through an application of association he will learn that $6=5+1=(4+1)+1$. It will be helpful also for him to realize that 4×567 is really nothing more than $(4\times500)+(4\times60)+(4\times7)$. Every child will appreciate multiplication more if he understands that 12×13 is equivalent simply to $(10+2)\times(10+3)=10\times10+10\times3+2\times10+2\times3$ (cf. Roman calculation, as well as modern mechanical methods). The multiplication table itself, indeed, might even be fascinating if it could be viewed wholly and regarded experientially. It is of primary importance that the formation of such a table be understood as short (continuous) addition. (Consult table in next column.) Students should be urged to make multiplying second nature, like walking—even though the first steps in both instances will be awkward and troublesome. In seeing how the basic laws apply to numbers, the student will not only gain an appreciation of the laws themselves, but will also enjoy discovering different ways of using them (cf. the rule for casting out nines for checking multiplication).

Division, in turn, should be approached as short (continuous) subtraction. In this connection interest may be aroused in

Multiplication Table
(Decimal)

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

prime numbers, i.e., a number that cannot be divided by any number except unity and itself (cf. Euclid's proof that there is an infinite number of primes). A student should certainly be introduced to convenient units of division, i.e., sub-divisions like 10 dimes in a dollar, and thus to the decimal system. Division of 530 by 10 should be recognized as being equivalent to division of 500 by 10 plus division of 30 by 10.

The general usefulness of the basic laws is largely responsible for the introduction of imaginative concepts throughout the history of arithmetic in order to preserve these very laws. Starting with the basic laws for positive integers one finds them being made requisite also for negative integers, then for fractions (hence all rational numbers), later for irrational numbers (every teacher should know Euclid's proof that $\sqrt{2}$ is irrational), and even for complex numbers. For example, one cannot prove that $(-1)\times(-1)=+1$; it must be postulated in order that the law of distribution for multiplication should be applicable also to negative integers.

On the other hand, some items are quite arbitrary as, for example, the rule

of addition for fractions. The teacher should realize that $2/3+4/5$ could have been defined initially as

$$\frac{2+4}{3+5}$$

(as many a hopeful youngster would sometimes wish). Such a rule, however, was rejected because it would not be useful for ordinary measurements. A fraction is essentially just a symbol of a number pair (m/n) which satisfies both the general laws and certain special rules. A few words about fractions may not be out of place here. Certainly the etymological meaning of associated words would be helpful at the start, i.e., a fraction as a broken part or subdivision, denominator as a class name (like denomination), numerator as simply the number of elements in that class. Fractions, I believe, should be related more to the parts than to the whole so that the process itself becomes naturally associated with counting subdivisions (cf. coins, weights, and measures).

I would begin, therefore, with fractions having similar denominators, e.g., decimal fractions. (N.B. Not every fraction can be represented decimally—and every teacher should be able to prove this.) It might be instructive to start with translating $2+3=5$ into $20/10+30/10=50/10$. If one would then regard $4/2$ as counting the number of half subdivisions in 4, one might find $5/2$ more meaningful. The student should appreciate that $1/2$ of one is equivalent to $1/4$ of two (both being referred merely to different subdivisions) and that $2/3$ of 6 is equivalent to $2\times6/3$.

One can begin the multiplication of fractions with $.5\times.6=5/10\times6/10=30/100=.30$—a natural association with decimals. Next a sequence might be used such as:

$$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\,;\ \frac{3}{2}\times\frac{1}{2}=\frac{3}{4}\,;\ \frac{3}{2}\times\frac{5}{4}=\frac{15}{8}.$$

Division, too, offers pedagogical challenges. For example, $12/2$ signifies merely the number of two's in 12—likewise $8/12$ divided by $2/12$ gives the number of $2/12$ in $8/12$, which is obviously 4. The student should realize that 6 divided by $2/3$ is the same as $(6\times3)/3$ divided by $2/3$, and hence equivalent to 6 multiplied by 3 and then divided by 2. (The advisability of a common denominator is evident here.) I may seem to have belabored fractions unduly. Perhaps, the awe of elementary pupils and the ignorance of college students with respect to them justify any attempt to urge improved understanding of fractions by students—and by teachers. Some years ago 85% of a large college course of mine in general physics got the wrong answer to the problem $1+1=1/x$. It has been my experience that three answers, viz., 1/2, 1, 2, will be readily obtained in any college class. (Try it in your own.) I once told the Dean of a Pharmacy school about this incident during a luncheon conversation. As we were leaving the table, he said, "What is the correct answer?"

EDITOR'S NOTE. There is a wide range of mathematical background between the typical elementary school teacher and the college professor of mathematics. It cannot be otherwise in a system where the elementary school teacher must have some background and competence in many different areas of learning and at the same time understand a good deal about children and how they learn and also about school dogma. Dr. Seeger writes from the point of view of a mathematician who is interested in what and how children learn. He has had many students who failed to think and to understand and he is concerned. It is our hope that the newer generation of pupils who are being taught to think and to understand will do better when they go to college even though they have not been taught arithmetic from a mathematician's point of view. It is sad that we have many teachers entering the profession without ever having had good background courses in mathematics. It is also sad that we have comparatively few people in the country who have a competence in both the mathematics and the teaching thereof for the range kindergarten through college.

Group Methods in Primary Grades

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EVERY TEACHER TRIES to give students as much individual attention as possible. But there is always the time problem, especially if classes are large.

Many teachers have found that organizing the class into several ability groups makes it possible to do a great deal to meet individual needs. Certainly assignments can be differentiated, with the advanced group covering sections which the average or slow group may omit.

At the risk of annoying the reader with a truism, our children vary widely in their ability to learn. Research shows that a range of FIVE years in achievement is not uncommon in any single grade.

Our teachers accept the fact that the children they teach vary widely. This, of course, is nicely illustrated in the field of reading. A conscientious teacher studies the children and the records which the preceding teacher has sent. She will have several conferences with the preceding teacher better to acquaint herself with the group. Then she will form the children into as many groups as seem necessary.

If this is a most desirable practice, and I believe it is, can any teacher make a valid distinction between the teaching of reading and the teaching of arithmetic?

Group for the Job at Hand

Basically we must begin every child where he is, not where some arbitrary grade organization or course of study places him. To do this effectively, we group for *a job*, or a felt *need*, or an *interest*. Many teachers provide in their daily plans a *SKILLS* period in which small groups are working on a variety of jobs. During this skills period the teacher moves from one group to another, guiding the groups toward their respective goals.

As teachers we know how indispensable it is to develop meanings, the "big ideas" (concepts) in elementary mathematics. You have heard for years that skills can be used intelligently only when they have been acquired intelligently. Teaching mathematics, elementary or college or graduate, meaningfully facilitates learning. To teachers everywhere that is a fundamental truth.

Another fundamental truth accepted without question is that in primary numbers work we should, nay must, employ *lots and lots and lots of counting devices.* (Beads, buttons, etc.) These things, which can be thrown away, are better than counting on fingers. And if we must say so, using fingers is better than rote memory.

Another illustration of a fundamental truth planned and practiced very well every school day by our primary teachers is the manner in which our teachers introduce little children to numbers and explaining their usefulness. Here the approach is varied. To mention a few:

1. Real situations—art work, games, banking, parties, projects and classroom organization and housekeeping.
2. Pictures and number illustrations—charts and illustrations made by the class.
3. Flash cards—each child should have his own set.
4. Individual appraisals—Teacher-pupil conferences.
5. New number concepts—learned by using facts previously studied.
6. Group work—to encourage thinking to help children learn skills better, to make it possible for each child to work at his own level and to contribute effectively, and to furnish opportunities for social development. This technic we must exploit.

7. Children's experiences in and out of school—playing with blocks, dramatized play, building and furnishing a play house or store or post office, playing games, distributing supplies, checking attendance, etc.

If grouping in arithmetic can be as readily accepted as grouping in reading, we shall find classrooms engaged in meaningful work for all its members. Can you imagine the wholesome classroom climate that would be created when every child has work which he can do successfully and in which he can see that he is making progress!

Teachers should not become discouraged if they seem to be groping when grouping. Planning and evaluating with each child in mind exceeds by far the easier method of gearing teaching to the non-existing average student. After all, in teaching arithmetic we only have three (3) things to do, 1) combine, 2) separate and 3) compare, which, reduced to its simplest term, is nothing more than a continuous grouping and regrouping of numbers.

Each Child Must Learn

Children must do the learning for themselves. All the teacher can do is stimulate and guide their learning. With this in mind several principles of planning come to mind about grouping:

1. There must be a real reason for grouping.
2. Groups are not rigidly fixed—a child can move about according to his needs and his interests. He may work in a certain arithmetic group, a different group in reading, the whole class in music, and still a different group in science. You will find that organizing your classroom into small groups will give a lift to teaching and to learning.
3. Pupils should understand the role of the teacher in the pattern of small groups. The teacher acts as a sharer and seeker of information and ideas.
4. Children should understand the goal of each group.
5. The goal of each small group should be related to a larger classroom goal.
6. The teacher takes time to confer with each group.
7. The teacher knows that the best basis for grouping children for arithmetic or any other subject is the teacher's judgment plus the results of standardized tests. Only one or the other method is often not enough.

Dr. Alice Miel, Associate Professor of Education, Teachers College, Columbia University, said: "The teacher must do enough preplanning but not too much. The teacher must give enough guidance but must not over direct. Caution and daring must be kept in balance—the teacher must say 'yes' a great deal, yet help the group sometimes to say 'no' to its own proposal. The teacher must also work hard to have the individual feel responsible for giving his ideas to the group and for respecting his own ideas."

When a child's interest is up, his resistance is down. In a book entitled "Teaching in the Small Community," which is the 1956 Yearbook of the National Education Association, I read a paraphrase on the story of Goldilocks: For each child, whether in a small community or large—An atmosphere not too hot, not too cold, but just right. A learning task—not too hard, not too soft, but just right. Achievement goals—not too high, not too low, but just right.

Our teachers can feel very pleased, very proud, and very confident that the teaching and learning of arithmetic today has advanced greatly over that of previous years.

EDITOR'S NOTE. The acquisition of learning and understanding is an individual job for each pupil even though he is aided by a teacher, a book, or other pupils. Since we cannot provide a teacher or tutor for each child or for each small group we do the next best thing, we organize groups within our classes. But these groups should be fluid. As Mr. Greco points out, a child should find a group where his individual needs can be met.

The Role of Experiences in Arithmetic

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IN EVERY MODERN ELEMENTARY SCHOOL, there are many natural situations which provide opportunities for children to think about and with which to use numbers. Classroom and school experiences are important because they pave the way for planning a program of meaningful mathematics.

Teachers welcome suggestions which help them in recognizing and utilizing experiences effectively in setting up situations which create a stimulating learning environment. The following typical classroom experiences were worked out in Assistant Superintendent Johanna Hopkins' districts in New York City. They are rich in both mathematical meanings and social significance. They afford practice for children to solve everyday problems so that arithmetic may be viewed not merely as a school subject but as a valuable part of out-of-school life as well.

Our Class Has a Party

Miss S. planned an experience with her pupils who were looking forward to an Easter party. Instead of buying all the goodies, they wanted to do something different like making chocolate pudding. They decided to use "Instant Pudding" since that would not involve the use of the school cooking room. Plans had to be made, decisions had to be reached, and rules had to be established. The date of the party was set for the day before Good Friday. The children had to consult the calendar and determine the date in April. They found how many days they had to wait for the party. As plans were formulated Miss S. wrote them on charts which she used for experiental reading as well as mathematics. Samples of these charts follow.

CHART 2

OUR RECIPE

1 package of pudding
2 cups of milk
This recipe makes enough for 4 children.

CHART 4

WHAT WE LEARNED

There are 9 groups of 4 in a class of 36.
We need 9 boxes of pudding.
There are 18 groups of 2.
We need 18 cups of milk.
We measured the number of cups in a quart. 4 cups of milk fill a quart. 18 cups fill 4 and one-half quarts. We found this out by pouring 18 cups of water into quart bottles.

CHART 3

WE PLAN TOGETHER

There are 36 in our class—35 children and one teacher.
We will stand in groups of 4 to find how many boxes of pudding we will need.
We will stand in groups of 2 to find how many cups of milk we will need.
We will find out how many quarts of milk we will need.

CHART 5

OUR JOBS

Jon, Tom, Jack, May, Ned, Ann, Sue, Bob, Mary	*will bring the 9 boxes of pudding.*
——, ——, ——, —— and ——	*will bring the quarts of milk.*
——, ——, ——, ——, ——, ——, ——, ——, ——	*will bring the 9 mixing bowls needed.*
——, ——, ——, ——, ——, ——, ——, ——, ——	*will bring the 9 egg beaters needed.*

Each child will bring his own dish and spoon.

A vital activity of this type is a cooperative enterprise in which all children participate. Their interest and curiosity are aroused so that they themselves seek answers to problems involving materials with which they are dealing. Mathematics becomes alive—a necessary tool of action.

Opportunities afforded for personal and social growth are innumerable because each child becomes a member of an active, functioning group.

EDITOR'S NOTE. Perhaps the best thing about an "arithmetic experience" is the opportunity to tie the arithmetic to a situation in which children are greatly interested and hence to whet their interests. The amount of arithmetic learned depends in large part upon the way in which the teacher provides for thinking and learning of essential arithmetical items. "It does something" to children when they feel a sense of responsibility in planning and in carrying out a project. Active learning seems to be more real and apparently is retained longer than passive learning in which a child learns just to satisfy a request or an assignment. The wise teacher will recognize and make use of the psychological values in pupil participation and at the same time keep her basic goals for the children firmly in mind.

BOOK REVIEWS

General Mental Functions Associated With Division by Henry Van Engen and E. Glenadine Gibb. 181 pp. Iowa State Teachers College, Cedar Falls, Iowa 1956. Price $2.00.

Here is a research report that should be read by all serious students of arithmetic. Basically it is an experimental study of learning division by two methods: (1) the conventional algorism and (2) a subtractive method. The treatment is descriptive and statistical with the usual measures of significance of differences of the groups compared. But this report of the research is much more than a simple statistical study: it contains a good deal of reflection upon educational doctrines, principles of learning, and implications of methods of teaching and learning. In fact, much of the worth of the report to many people will be in the discussions of the mental functions associated with division as the title suggests.

The table of contents, reproduced in part below, shows the scope of the study.

Chapter I. Historical Background
Chapter II. Division as Successive Subtraction
Chapter III. Some Psychological Considerations
Chapter IV. The Plan of the Study
Chapter VI. A Descriptive Analysis of the Findings

Chapter X. Summary and Conclusions
Chapter XI. Implications

Teachers and curriculum workers generally should read Chapters I, II, III, IV, X, and XI for the educational values discussed. Most of the other chapters are concerned with statistical analysis which should certainly be studied carefully by those competent in statistical research.

Doctors Van Engen and Gibb should be highly commended for their attempt to carry on a study that is concerned with mental processes involved in learning division and to sense division situations and the uses of the process. This is far different from the older studies that were concerned with more and more minute analysis of the component skills in the division process.

The findings of the study should be read in the original report. They become more significant in terms of a framework of one's basic vision of the psychology of learning particularly if one has accepted an empirical "field theory." The final statement of the authors is significant; " . . . if the school is to achieve the broad general objectives it so frequently claims to achieve, this study has shown that arithmetic should not be analyzed in terms of the computational processes. Arithmetic is more than its end result. It is a conceptual process, and conceptual processes are not susceptible to unitary analysis."

BEN A. SUELTZ

A New Look at the Fractions by V. P. Palmer. 65 pp. V. P. Palmer, 1322 Twelfth St., Rockford, Ill. 1956. Price $2.95.

In announcing this book which is enlivened with several interesting cartoons the author states in part: " . . . A new book that will either be a boon to education or set it back fifty years." Probably neither will result. In essence this book gives methods of remembering how to convert the most commonly used common fractions to their decimal equivalents. This is done by indirect association with a "key word." For example, the "ATE" fractions are so named because "ate" sound like "eight" and ate has three letters and there are three decimal places when common-fraction eighths are converted to decimal fractions. The scheme is like the one that many people learned in Sunday School for remembering the number of books in the bible. The "key number" is three. Then $3 \times 3 = 9$, combining 3 and 9 for 39 gives the number of books in the Old Testament; and $3 \times 9 = 27$, which is the number of books in the New Testament.

Rules of association such as these help many people to remember. Likewise repeated use aids both in remembering the rule and the result. Certain workers in industrial plants may find this book valuable if their jobs call for repeated use of conversion from common to decimal fractions. Many such workers now use tables, others have memorized the equivalents which they most frequently use. In the schools, we want to use methods that have meaning within the mathematics of the work. Unless we drop all computational work with common fractions and use their decimal equivalents it is doubtful if we will have sufficient use for learning indirect mental modes of conversion to warrant spending much time learning the procedure. However, this is an interesting idea that will appeal to some junior high school pupils whom all of us have met.

BEN A. SUELTZ

Stimulating the Better Arithmetic Pupil

With Insight Producing Activities

A. R. GASKILL

Central Michigan College, Mount Pleasant, Michigan

DID YOU KNOW that there is currently in use a number system which has only two symbols (or digits) instead of the ten (0–1–2–3–4–5–6–7–8–9) that we customarily use? Many of our "mechanical brains" find this system very convenient. Perhaps you have already heard of this way of writing numbers. It is called the binary (or base-two) system. Would you like to learn how it works?

To understand (and use) the binary system we need keep two things in mind: first, that we use two symbols only ("0" and "1"); and second, that instead of ten as the base of the number system we use two. By this we mean that each position to the left of the units place is a power of two. To illustrate, in the first, or units position we write either "0" or "1." (Since these are the only digits available to us.) In the next place to the left we again write either "0" or "1," but in this instance we mean either no or one "two." In the next place to the left the "1" would mean one group of four units, and so forth. See the illustration which follows.

Place Value in the Binary System

n	f	e	d	c	b	a
etc.	32	16	8	4	2	1

Thus, to write the number three, we would write "1" in the units place, column "a" (meaning one *one*), and then write "1" in column "b" meaning one *two*. Our three, consequently, in the binary system would be written "11," or one *one* plus one *two*. Let us look at the following diagram and see how some other numbers are written in this system.

Selected Base-Ten Numbers Written as Binary Numbers

	g 64	f 32	e 16	d 8	c 4	b 2	a 1	Base-Ten Equivalents
w			1	0	1	1	0	22
x	1	0	0	0	1	1	1	71
y		1	1	1	0	0	1	?
z				1	1	1	0	?

Line "w," for example, shows one group containing sixteen units, no group of eight units each, etc., etc. Thus, the number twenty-two (22) in the base-ten system would be written as 10110 in the binary system. You will note that the zero serves as a place holder in both systems.

The next line, "x," tells us that we have one group of sixty-four units, one group that has four units in it, one group having two units in it, plus one more unit for a total of seventy-one units. Can you read lines "y" and "z" without help? Try these on your boys and girls. They will enjoy them. What is the equivalent of the binary number 110011101 in the base-ten number system? You will need to extend the diagram shown to arrive at this answer.

Perhaps some pupils would like to investigate and report upon uses of the binary system. Others may wish to make up problems using this base-two system. The possibilities are numerous.

Now that we have the binary system well in hand, let's try something a little different, yet similar. Instead of using a system which has only two digits, and based on a place value of two, let's use a different number base. How about six?

This means that we will need six symbols as follows (0–1–2–3–4–5). We will see how to write larger numbers in a moment.

In this system each place value to the left of the units position will be a power of six. (See diagram which follows.) Numbers one through five are written in the traditional way, and in their customary position.

Selected Base-Ten Numbers Converted into Their Base-Six Equivalents

	e 1296	d 216	c 36	b 6	a 1	Base-Ten Numbers
tt				1	0	6
uu				5	2	32
vv			1	1	0	42
ww		5	0	3	2	1100
xx			4	4	4	??
yy		3	2	5	3	753
zz						1957

To write the number six we put a "1" in column "b" and a zero as a place holder in column "a." Thus, six would be written "10" in this base-six system. (See diagram, line "tt.")

Illustrating again, we would write thirty-two by placing a "5" in column "b" and a "2" in column "a." (See line "uu".) Line "vv" shows one group of thirty-six units, one group of six units, plus no additional units or a total of forty-two units.

Line "ww" includes five groups of two hundred sixteen units each (1080 units in a base-ten system); three groups of six in each group, or a total of eighteen; and two units, for a grand total of eleven hundred units. That is to say, 1100 in a base-ten system would be written 5032 in a number system which has six as its base. What is the base-ten equivalent for line "xx"?

To achieve added insight, we might ask ourselves how we would write the number seven hundred fifty-three from our base-ten system into a system which uses six as its base. To do this we proceed as follows. By referring to our last diagram we note that the number we wish to write falls between 216 and 1296. We ask ourselves how many 216's there are in 753. Since three is correct, we write this digit in column "d." This leaves a remainder of 105. We then ask how many 36's there are in 105. Two is correct, so this digit goes into column "c." With a remainder of thirty-three we see that we should enter "5" in column "b," which leaves three units (or the digit "3") to be shown in column "a." Thus our number reads "3253." In the base-six system this number is the same as "753" in the base-ten system. Can you write the calendar year 1957 using the base-six system? Use line "zz" which has been left blank for you.

Would you like to experiment with other number bases? How about a system which would use a number larger than ten as its base? How would you do that? Stimulate your better pupils with this challenge. They will have fun, and learn.

EDITOR'S NOTE. The introduction of a base other than ten is intriguing to a number of pupils. The mechanics of changing from one base to another should be accompanied with reasoning and understanding. Also, pupils in the upper grades might well explore some of the reasons why certain bases other than ten have particular values, as for example, the base eight makes it possible to change our most commonly used fractions to a simple "point notation" and thereby greatly simplify computations. Likewise in a base twelve it would be possible to compute directly with feet and inches in "point notation." How much of this should be done in the elementary school depends in large measure upon the pupils and their teacher.

Report of the Nominating Committee

The Committee on Nominations and Elections presents below the names of persons nominated for offices to be filled in the election of 1957. The term of office for the Vice-Presidents is two years. Three directors are to be elected for terms of three years.

Attention is called to the following provisions adopted by the board of directors.

"Nominations shall be made so that there shall be not more than one director elected from each state, and that there shall be one director, and not more than two, elected from each area." For a map of the regions as now constituted, members may consult THE MATHEMATICS TEACHER for October, 1955, page 442. Under the regulation quoted above it is necessary to insure the election of at least one director each from the Western and Southeastern Regions. To avoid a situation in which nomination would be equivalent to election, the Committee has nominated two candidates from the Western Region and two candidates from the Southeastern Region, and has adopted the following rule for determining who shall be declared elected as directors:

The three candidates receiving the largest number of votes will be declared elected, provided the Western and Southeastern Regions are each represented among these three candidates. If either of these two regions should not be so represented, then the candidate, within that region, receiving the larger number of votes will be declared among those elected.

Ballots will be mailed on or before February 6, 1957 from the Washington office to members of record as of that date. Ballots returned and postmarked not later than March 6, 1957 will be counted.

The Committee wishes to thank all those who suggested persons for consideration and urges all members to vote.

F. LYNWOOD WREN, Chairman
ALICE M. HACH
MAURICE HARTUNG
JANET HEIGHT
CLARK LAY
ZEKE LOFLIN
JOHN MAYOR
BESS PATTON
PHILIP PEAK

NOMINEES FOR VICE-PRESIDENT—COLLEGE LEVEL

ROBERT E. PINGRY

HENRY W. SYER

Robert E. Pingry

Associate Professor of Mathematics and of Education, University of Illinois.

A.B. and A.M., Ball State Teachers College; Ph.D., Cornell University.

Teacher of Mathematics, Mitchell, Indiana High School; Teacher of Physics, Chemistry, and Mathematics, Griffith, Indiana High School and Junior High School; Instructor in Mathematics and Navigation, U.S. Naval Reserve Midshipman's School, Cornell University; Navigator of an aircraft carrier on Pacific duty; Instructor in Mathematics, Bowling Green State University; Teaching Assistant in Mathematics, Cornell University; Visiting Lecturer in Education and in Mathematics, University of Colorado.

Member: NCTM, Mathematical Association of America, Illinois Council of Teachers of Mathematics, Phi Delta Kappa, Phi Kappa Phi, Kappa Delta Pi, AAUP.

Activities: Member Joint Committee of NCTM and MAA on teacher education in mathematics, 1953–54. Member of NCTM committee on research in algebra, 1953–55. Editor of *Newsletter* of Illinois Council of Teachers of Mathematics. Member of Executive Board of Illinois Council of Teachers of Mathematics. Member of Committee on Contests and Awards of the Illinois Section of the Mathematical Association of America. Numerous speeches and papers at mathematics conferences, teachers' institutes, and National Council meetings.

Publications: Coauthor, "Problem Solving in Mathematics," Chapter 8, *Twenty-First Yearbook* of NCTM. Coauthor of a series of junior high school textbooks in general mathematics, *Using Mathematics*, 7–9. Articles in THE MATHEMATICS TEACHER, *Journal of Experimental Education*, and *The Education Digest*.

Henry W. Syer

Associate Professor of Education, Boston University, Boston, Massachusetts. In charge of Teaching of Mathematics, Audio-Visual Education, and Director of Travel Courses.

B.S., M.A. and Ed. D., Harvard University.

Instructor in mathematics and science, Gunnery School, Washington, Connecticut; Instructor in mathematics, Culver Military Academy, Culver, Indiana; U. S. Army, Anti-aircraft Artillery (Computer Section) and Signal Corps (Photographic Section); Assistant and associate professor of education, School of Education, Boston University; Visiting professor, School of Education University of Michigan.

Member: NCTM, Association of Teachers of Mathematics in New England, NEA, AAAS, MAA, Phi Beta Kappa, Phi Delta Kappa. Listed in: *Leaders in Education*, *Who's Who in Education*, and *Who's Who in the East*.

Activities in NCTM: Coeditor, "Aids to Teaching" Department, THE MATHEMATICS TEACHER, 1948–1954; Committee on Evaluation of Films and Filmstrips, 1948–1957 (Chairman, 1948–1951); Committee on Institutes, Workshops, and Conferences, 1948–1957; Board of Directors, 1949–1952; Associate editor, THE MATHEMATICS TEACHER, 1950–1952; Chairman, Committee on Supplementary Publications, 1950–1955 (Called "Committee on Publications of Current Interest," 1950–1953); Committee on Evaluation and Research, 1951–1953; Program Committee, 12th Summer Meeting, held jointly with N.E. Institute, 1952; Committee on Materials, Summer Meeting, 1954; Yearbook Planning Committee for "Basic Concepts in Mathematics," 1954–1957; Committee on International Cooperation in Mathematics, 1954–1957 (Chairman, 1956–1957); Co-operative Committee on Science and Mathematics of the AAAS, 1954–1957; Planning Committee for Local Arrangements, Annual convention, Boston, 1955; Participant in conventions (speaker, leader of film forums, demonstration lesson, science demonstration, etc.), 1938–1957.

Other Activities: in Association of Teachers of Mathematics in New England: Organizer and first general chairman, 1949; Member of Council, 1950–1955; Chairman, Committee on Publicity and Publications, 1950; Committee on Mathematical Games and Puzzles, 1950–1951 (Chairman, 1951); Program Committee, 1952–1955; Mathematics Contest Committee, 1952–1953; Vice-President, 1954–1957; Chairman, Committee on Membership and Hospitality, 1956–1957. Other activities: Consultant to Commission on Motion Pictures, American Council on Education, 1947–1948; Committee on Mathematics for Journal of American Association of Colleges of Teacher Education, 1953; United States sub-committee of International Instruction Committee for Mathematics of International Union of Mathematics, 1952–1955; Committee on National Mass Media Awards for Thomas Alva Edison Foundation, 1955; Committee on Recruitment and Guidance Material for Science and Mathematics, AAAS, 1955–1957; Representative, Council on Student Travel, 1955–1957; Examiner in Audio-Visual Education, Massachusetts Civil Service, 1951–1955; Tour leader, Boston University Travel Courses, 1953 (Mathematics Tour), 1955; European Coordinator, Boston University Travel Courses, 1956; Curriculum consultant in mathematics; Consultant on filmstrips.

Publications: Numerous articles in: *School Science and Mathematics*, THE MATHEMATICS TEACHER, *The Philosophical Forum*, *Film News*, *Audio-Visual Guide*, and chapters in *Yearbooks* of NCTM.

NOMINEES FOR VICE-PRESIDENT—JUNIOR HIGH SCHOOL LEVEL

ALICE M. HACH

ELIZABETH JEAN ROUDEBUSH

Alice M. Hach

Consultant in Mathematics, Racine Public Schools.

B.A., University of Iowa; M.A., University of Michigan.

Elizabeth Jean Roudebush

Teacher, Junior High School Mathematics, Stuart, Iowa, Fort Dodge, Iowa, Racine, Wisconsin, Ann Arbor, Michigan; Critic teacher, University High School, Ann Arbor, Michigan; Consultant in Mathematics, Racine Public Schools.

Member: NCTM; CASMT; NEA; Wisconsin Education Association; Michigan Council of Teachers of Mathematics; Women's Mathematics Club of Chicago and Vicinity; Association of Childhood Education; Wisconsin Mathematics Council; Pi Lambda Theta; Delta Kappa Gamma.

Activities in NCTM: Member of Place of Meetings Committee; Editorial Board of forthcoming Yearbook; Cochairman for local arrangements of summer meeting of NCTM, Kalamazoo, Michigan, 1953; Committee member of the mathematics number of the *National Association of Secondary School Principals Bulletin*, May, 1954; Cochairman for local arrangements of annual meeting of NCTM, Milwaukee, 1955; Appearance on convention programs.

Other activities: President and Vice-President of Iowa Mathematics Association; Member of Curriculum Committee for State of Iowa; Chairman of Mathematics Section for Michigan Schoolmasters' Club; Member of Board of Directors for Michigan Council of Teachers of Mathematics; Chairman of Mathematics Section for Michigan State Association; Director of Arithmetic Summer Workshops, Eastern Oregon College of Education, La-Grande, 1952, 1953; Oregon College of Education, Monmouth, 1954; Staff Member of Summer Institutes, Mathematics Teachers of New Jersey, Rutgers University, 1955, New England Teachers of Mathematics, Williams College, 1956; President of Pi Lambda Theta, Beta Chapter; Member of Resolutions Committee for CASMT, Certificate Award from Quiz Kids for being one of most popular classroom teachers in nation; Summer Camp Conservation Scholarship, Ann Arbor Michigan Garden Club; Treasurer of Delta Kappa Gamma —Beta Chapter; Listed in *Who's Who in American Education.*

Publications: *Twenty-Second Yearbook of NCTM*: "Reporting to Parents Pupil Progress," "Importance of Early Guidance at Junior High School Level"; THE MATHEMATICS TEACHER: "A Project Giving Practice in Reading and Writing Roman Numerals," "As Others Saw Him"—The Schorling Number; *Junior Scholastic*: "Junior Scholastic Brings a Fresh Approach to Problem Solving."

Elizabeth Jean Roudebush

Director of Mathematics from Kindergarten through Grade Twelve for Seattle Public Schools, Seattle, Washington.

Graduate of State College of Washington, Pullman, Washington: A. M., Teachers College, Columbia University; summer sessions at University of Washington, University of California at Los Angeles, and University of Southern California.

Teacher, Mathematics, Roosevelt High School, Seattle, Washington; Mathematics Department Head, Edison Technical School, Seattle, Washington; Director of Mathematics, Seattle, 1949—. During World War II served in WAVES as Women's Reserve Representative at U. S. Naval Hospital in Chelsea, Massachusetts.

Member: NCTM; ASCD; NEA; Washington Education Association (Past President of local group affiliated with WEA and NEA); Delta Kappa Gamma (Past Vice-President of local chapter); Pi Lambda Theta (Past President of local alumnae chapter).

Activities in NCTM: Member, Board of Directors, 1953–56; Regional Representative of Western Region of Affiliated Groups; Chairman of Place of Meetings Committee; Cochairman in Charge of Local Arrangements for 1954 Summer Meeting; Section Speaker at 1953 Christmas Meeting; Chairman of Affiliated Groups 1956–59.

Other Activities: helped organize Puget Sound Council of Teachers of Mathematics; helped organize Washington State Mathematics Council.

Publications: *Laboratory Geometry*; "An Arithmetic Bulletin for Parents," THE MATHEMATICS TEACHER, May, 1951.

NOMINEES FOR THE BOARD OF DIRECTORS

CLIFFORD BELL

SYLVIA VOPNI

ROBERT E. K. ROURKE

HELEN L. GARSTENS

ANNIE JOHN WILLIAMS

LOTTCHEN LIPP HUNTER

Clifford Bell

Western Region

Professor of Mathematics and Head of Mathematics Extension, Los Angeles Campus, University of California.

B.S. and Ph.D., University of California, Berkeley.

Teaching Fellow, University of California Berkeley; Instructor to Professor, University of California, Los Angeles; Educational Supervisor in Charge of Mathematics Courses, Southern California area, University of California ESMWT program. In charge of teacher-training courses in Department of Mathematics, University of California, Los Angeles.

Member: NCTM, California Mathematics Council, NEA, MAA, AMS, AAAS, and a number of honor societies. Listed in *American Men of Science, Who's Who in*

Education and other biographical books.

Activities in NCTM: Membership Committee; Institute Committee; Joint Committee on Co-operation with Industry and the later Committee on Co-operation with Industry; Curriculum Committee; Publications Board; Nomination and Election Committee (1955); Chairman, Local Arrangements Committee, Fourteenth Christmas Meeting in Los Angeles; Co-chairman, Sixteenth Summer Meeting in Los Angeles.

Other activities: Chairman of the Planning Committee of the Annual California Conference for Teachers of Mathematics and Director of the Conference.

Coauthor of textbooks in trigonometry, commercial algebra, and the mathematics of finance. Author of articles in the *Bulletin* of the American Mathematical Society, *The American Mathematical Monthly*, THE MATHEMATICS TEACHER, *The Arithmetic Teacher*, other national and foreign journals.

Sylvia Vopni

Western Region

Assistant Professor of Education, University of Washington, Seattle, Washington.

B.A., M.A., Ph.D., University of Washington.

Teacher of mathematics and science, public schools of Washington State; physicist, Navy Department; teacher of mathematics and physics, Edison Technical School; Northwest Regional Soroptimist Fellowship in Education; head of department of mathematics and science, Edison Technical School; Assistant Researcher in Education, University of Washington, also teaching summer school and extension division classes in education and mathematics; Acting Assistant Professor of Education.

Member: NCTM; MAA; AAAS; American Statistical Association; Washington Council of Mathematics Teachers; NEA; ASCD; Washington Education Association; American Association of University Women; Phi Beta Kappa; Pi Lambda Theta; Delta Kappa Gamma; Pi Mu Epsilon.

Activities in NCTM: Cochairman on Local Arrangements, Fourteenth Summer Meeting, 1954; discussion leader or participant on NCTM programs, 1949, 1953, 1954, 1956; member of committees on television, scholarships, and summer conferences. Western Vice-President, Washington Council of Mathematics Teachers, 1955– .

Other Activities: President, Seattle Association of Classroom Teachers; president, Omicron Chapter, Delta Kappa Gamma; national vice-president, Pi Lambda Theta. Participant in several mathematics conferences and summer institutes.

Publications: Associate editor, *Pi Lambda Theta Journal*, 1946–49; feature editor, *Educational Horizons*, 1954–55; "Nation's Mathematics Teachers Coming West." *Washington Education*, 1954; "Resources in the Teaching of Elementary School Science." *College of Education Record*, March, 1955; "Mathematics and the Humanities," *College of Education Record*, May, 1955.

Robert E. K. Rourke

Northeastern Division

Head of Department of Mathematics, Kent School, Kent, Connecticut.

Honour B.A., Queen's University, Kingston, Ontario; A.M., Harvard; course work for doctorate in mathematics, Harvard.

Teacher, Head of Mathematics Department, Headmaster, Pickering College, Newmarket, Ontario.

Activities: Past President of Ontario Association of Teachers of Mathematics and Physics; former executive of Canadian Mathematical Congress; member of Commission on Mathematics of CEEB.

Publications: Coauthor of *An Advanced Course in Algebra; Trigonometry and Statistics; Mathematics for Canadians*, Books I, II, III; articles in professional periodicals.

Helen L. Garstens

Southeastern Region

Assistant in Charge of Mathematics, in Office of Secondary Supervision, in Arlington County, Virginia.

Graduate of Hunter College; postgraduate work in mathematics at Columbia University; additional summer study at University of Virginia, Georgetown University.

Teacher, elementary schools and secondary schools of Arlington County, secondary schools of New York City, and at Hunter College and Brooklyn College in New York City.

Member: AEA, VEA, NEA, NCTM, Theta Mu Tau, Pi Mu Epsilon, Phi Beta Kappa.

Activities: Instrumental in organizing an Arlington Group of NCTM; Chairman of Arlington Group, responsible for affiliation, arranging programs, etc.; served on Advisory Board for Washington meeting of NCTM in December 1955. Present position involves providing leadership and active participation in curriculum development, in organizing in-service programs for professional improvement, in providing assistance to teachers of secondary mathematics in all phases of classroom activity. Worked closely with the National Science Foundation in Washington D.C. and on the Scholarship Committee of Arlington County Schools, in developing and establishing the "Arlington Plan" of scholarships to teachers for advanced study.

Annie John Williams

Southeastern Region

Teacher of Mathematics, Durham High School, Durham, North Carolina.

A.B., Greensboro College; M.A., University of North Carolina; additional graduate study at North Carolina State College and Duke University; attended Mathematics Institutes of Duke University (1944, 1946–52), New England (1952), and University of Virginia (1953–54); attended Laboratory Conference for Teachers of Science and Mathematics, Duke University, 1953–54, 1956 (Instructional Staff, 1956).

Teacher of Mathematics, Blackstone College, Blackstone, Virginia; Hoke High School, Raeford, North Carolina; Massey Hill High School, Fayetteville, North Carolina; Alexander Graham Junior High School, Fayetteville, North Carolina; Julian S. Carr Junior High School, Durham, North Carolina.

Member: NCTM; NEA; MAA; CASMT; North Carolina Education Association (President, Department of Mathematics, North Central District, 1949–50; State President, Department of Mathematics, 1954–55; Chairman, Program Committee, Mathematics Conference, University of North Carolina, 1954–56); Durham City Education Association (Treasurer, 1945–46); AAUW (Vice-President, Local Branch, 1946–48); Steward, Duke Memorial Methodist Church; Delta Kappa Gamma (Recording Secretary of Eta Chapter, 1956–).

Activities in NCTM: North Carolina State Representative, 1947–54; Delegate to Delegate Assembly, 1953, 1955–56; Member, Agenda Planning Committee for the Seventh Delegate Assembly; Member, Committee on Relations with the NEA; appearance on convention programs.

Publications: "Organizing a Mathematics Club," THE MATHEMATICS TEACHER, February, 1956.

Lottchen Lipp Hunter

Southwestern Region

Curriculum Consultant for the public schools of Wichita and educational institutes in Kansas, and Head of Mathematics Department, Wichita High School West.

A.B. degree, magna cum laude, University of Wichita, M. A., Teachers College, Columbia University, Ph.D. Columbia University.

Teacher, junior and senior high schools of Wichita, Kansas; Summer sessions, University of New Mexico and University of Wichita.

Member: NCTM; Phi Lambda Theta; Kappa Delta Pi; American Association of University Women; National Society for the Study of Education; ASCD; NEA; KSTA; and the Wichita and Kansas Affiliated Groups of NTCM.

Activities: State President of the Kansas Association of Teachers of Mathematics and chairman for the local Affiliated Group.

Publications: Articles in the *Twenty-Second Yearbook* of the NCTM and *The Bulletin* of the Kansas Association of Teachers of Mathematics; curriculum pamphlets: *A Look around Wichita at the Use of Mathematics in the Home, Business, and Industry; Group Dynamics in Secondary School Mathematics; Understanding the Why and How of Mathematics through Things to Make and Do.*

Annual Business Meeting

Notice is hereby given, as required by the Bylaws, that the Annual Business Meeting of the National Council of Teachers of Mathematics will be held during the Thirty-Fifth Annual Meeting, March 28–30, 1957, at the Bellevue-Stratford Hotel, Philadelphia, Pennsylvania.

THE THIRTY-FIFTH ANNUAL CONVENTION NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Bellevue Stratford Hotel, Philadelphia, Pennsylvania

March 27–30, 1957

Twentieth century mathematics, and its implication for teaching mathematics from grade one through the junior college will be highlighted at the Thirty-fifth Annual Convention of the National Council of Teachers of Mathematics at Philadelphia. The current thinking and significant experimental and research study in modernizing our mathematics curriculum will be presented by outstanding leaders in mathematics and education.

Included among the special events is the opening address on Thursday evening by Dr. Mario Salvadori, Professor of Civil Engineering at Columbia University entitled "*The Universal Fear of Mathematics.*" At the General Session on Friday morning the new Twenty-third Yearbook of the Council will be presented. This book, written by thirteen mathematicians of the highest rank is entitled "*Insights into Modern Mathematics.*" This book will be outlined and reviewed by Dr. Bruce Meserve of Montclair, New Jersey.

At the banquet on Friday night, an outstanding woman in the field of mathematics, Dr. Marguerite Lehr, of Bryn Mawr College will deliver the address with the intriguing title "*More Knowledge or More Knowing.*" The luncheon speaker on Saturday is the well-known popular Colonel Robert C. Yates of Virginia's William and Mary College, who will speak on "*The Student You Send to College.*"

On Thursday afternoon, Dr. Philip Jones of Ann Arbor will conduct a Student Hour for 600 high school juniors and seniors outstanding in their work in school mathematics. His title is "*Mathematical Wonders.*" Our Saturday morning session we are keeping as a surprise package which we hope will be "*A Delight and Prelude*" to all the sessions the rest of that day.

There will be book exhibits, commercial exhibits, student exhibits, and a trip to the famous Benjamin Franklin Institute of Science. The social events promise a gala time for accompanying wives, husbands, and friends whose interests are other than mathematics. The various section meetings from Thursday morning to late Saturday afternoon feature the men and women who have significant and practical contributions to bring to our profession.

Every elementary school, secondary school, and college should have at least one representative for their institution at this Convention.

Wednesday, March 27

Elementary and secondary schools in Philadelphia, having special programs or doing outstanding work in mathematics, will be open for visitation this entire day. In the evening, a visit to the Franklin Institute of Science and the Planetarium offers both entertainment and education. Come to the convention early and enjoy these events.

Thursday, March 28

The annual meeting of the Delegate Assembly convenes at 9:00 A.M. Visitors as well as official delegates to the convention are welcome to this three hour session. For those not attending the Assembly there are two sightseeing trips, one to historic Philadelphia, the other to Valley Forge, a showing of mathematical films, and also further opportunity to visit Philadelphia schools. Time has also been provided for meetings of the several committees of the Council.

Thursday, P.M.

From 2:00–3:45 P.M. the convention will get into real swing with six outstanding sectional meetings, one at each of the levels, elementary, junior high school, senior high school, and college, another on Research in Mathematical Education, and a special section, sponsored by the Association of Mathematics Teachers of New Jersey on "Space Perception." This section features Lt. Louis J. Cavello, U.S.M.C. Artillery School, Quantico, Virginia.

From 3:45 to 5:15 P.M., there are laboratory sections on the elementary, junior high school, and senior high school levels, and from 4:00 to 5:00 P.M. the high School Youth Forum.

The State Representatives of the NCTM will meet with Mr. Ahrendt, the Executive Secretary of NCTM and Miss Mary Rogers, chairman of the Membership Committee from 7:00 to 7:45 P.M.

At the General Session beginning at 8:00 P.M., Dr. Allen H. Wetter, Superintendent of Schools of Philadelphia, will give an official welcome to be followed by Dr. Salvadori's address. The evening concludes with a reception to all convention members given by the host organization, the Association of Teachers of Mathematics of Philadelphia and vicinity.

Friday, March 29

This is really modern mathematics day. After the opening session in which the Yearbook will be presented, there will be four large section meetings at each of the several grade levels. The elementary school section will feature Maurice Hartung of Chicago and Agnes Gunderson of Wyoming. The junior high school session has John Mayor and Alice Hach considering the mathematics improvement at this level. The senior high school session will feature modern concepts of Sets, Sentences and Relations, and their Implication for the Secondary School Program. The remaining section will consider Mathematics in General Education, especially the Role of Statistics and Mathematics in Everyday Life.

Friday, P.M.

The afternoon session from 2:00–3:45 P.M. breaks away from grade levels (horizontal sections) and provides vertical sections on the topics Evaluation and Testing, International Relations, The Continuous Curriculum, The Training of Teachers, and Enrichment, Clubs and Contests. There will also be a reshowing of mathematical films from 2:00–4:00 P.M.

The feature of the afternoon will be the Annual Business Meeting of the Council. This is an opportunity for every member to hear what the National Council is doing in Mathematics Education. The work of your Board of Directors, the work of the committees, the election results, the publications, and proposed projects of the Council will be presented. Members are free to raise questions, make proposals, and direct the thinking of the Board to matters of concern to the Council at large. We hope all members will attend.

The day's activities conclude with the Annual Banquet, always an outstanding and rewarding affair and with a real treat in store in the address of Dr. Marguerite Lehr.

Saturday, March 30

The opening session is planned for 9:00 A.M. with a mathematician as speaker who has gained world reknown for his popularization of mathematics. This address was planned for this session so that the many teachers who could not attend the meetings on Thursday and Friday would have the opportunity of hearing this speaker. Following his address, there will be four sections, one at each level of instruction, and a fifth section, sponsored by the Teachers of Philadelphia on "Planning the Mathematics for a Large City System." In these sessions, at your level of interest, you will hear of the leading movements in mathematical education both in our country and abroad.

The luncheon promises to be a gala affair with Mr. Linton of Philadelphia toastmastering and Colonel Yates lecturing in a fashion that is all his own.

Saturday, P.M.

The convention does not end until 3:45 P.M. From 2:15 to 3:45 we have arranged the most important sections of the convention. Each of these sections deals with "The *Teaching* of Mathematics" with concrete practical suggestions on content and methods. For grades 1 to 8, there is a section on "Specific Techniques for Improving Instruction." For the secondary school teachers there are two sections, one on specifics for the abler students, and one on specifics for the non-college bound students. At the college level, there is a section on specific content for the freshman year.

All members should plan on attending one of these sections.